Name:________________________________

Exam Number:_______

University of Michigan Physics Department
Graduate Qualifying Examination

Part I: Classical Physics
Saturday 10 January 2015 9:30 am – 2:30 pm

This is a closed book exam, but a number of useful quantities and formulas are provided in the front of the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it. Answer the questions directly in this exam booklet. If you need more space than there is under the problem, continue on the back of the page or on additional blank pages that the proctor will provide. <u>Please clearly indicate if you continue your answer on another page. Label additional blank pages with your exam number, found at the upper right of this page (but not with your name). Also clearly state the problem number and "page x of y" (if there is more than one additional page for a given question).</u>

You must answer the first 8 required questions and 2 of the 4 optional questions. Indicate which of the latter you wish us to grade (e.g. by circling the question number). We will only grade the indicated optional questions. Good luck!!

Some integrals and series expansions

$$\int_{-\infty}^{\infty} \exp(-\alpha x^2)\,\mathrm{d}x = \sqrt{\frac{\pi}{\alpha}}$$

$$\int_{-\infty}^{\infty} x^2 \exp(-\alpha x^2)\,\mathrm{d}x = \frac{1}{2}\sqrt{\frac{\pi}{\alpha^3}}$$

$$\exp(x) = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$$

$$\cos(x) = 1 - \frac{x^2}{2} + \frac{x^4}{4!} + \cdots$$

$$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots$$

$$(1+x)^{\alpha} = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \cdots$$

Some Fundamental Constants

speed of light $c = 2.998\times10^{8}$ m/s

proton charge $e = 1.602\times10^{-19}$ C

Planck's constant h = 6.626×10^{-34} J·s = 4.136×10^{-15} eV·s

Rydberg constant $R_{\infty} = 1.097\times10^{7}\,\mathrm{m}^{-1}$

Coulomb constant $k = (4\pi\varepsilon_0)^{-1} = 8.988\times10^{9}\ \mathrm{N{\cdot}m^2/C^2}$

vacuum permeability $\mu_0 = 4\pi\times10^{-7}$ T·m/A

universal gas constant $R = 8.3$ J / K·mol

Avogadro′s number $\mathrm{N_A} = 6.02\times10^{23}\,\mathrm{mol^{-1}}$

Boltzmann′s constant $\mathrm{k_B} = \mathrm{R/N_A} = 1.38\times10^{-23}$ J/K $= 8.617\times10^{-5}$ eV/K

Stefan-Boltzmann constant $\sigma = 5.67\times10^{-8}\,\mathrm{W/m^2K^4}$

radius of the sun $R_{\mathrm{sun}} = 6.96\times10^{8}$ m

radius of the earth $R_{\mathrm{earth}} = 6.37\times10^{6}$ m

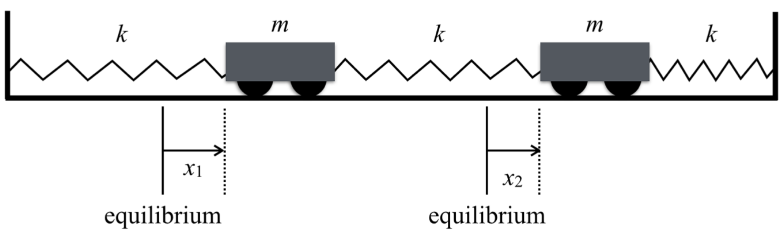
radius of the moon $R_{\mathrm{moon}} = 1.74\times10^{6}$ m

gravitational constant $G = 6.67\times10^{-11}\,\mathrm{m^3/(kg{\cdot}s^2)}$

1. Classical Mechanics:
Two identical carts of mass m move without friction on a horizontal track, between two fixed walls as shown in the figure below. Each cart is attached to its adjacent wall by a massless spring with spring constant k, and the carts are also attached to each other by a massless spring with spring constant k. The carts' positions x_1 and x_2 are measured from their respective equilibrium positions.

a) Derive the equations or motion for the system. Express them in matrix form $\underline{\underline{M}}\ddot{\vec{x}} = -\underline{\underline{K}}\vec{x}$. Here $\vec{x}$ represents a vector of displacements $\vec{x} \equiv \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$, and $\underline{\underline{M}}$ and $\underline{\underline{K}}$ are matrices.

b) Calculate the normal frequencies and the corresponding normal modes for the system.



2. Thermodynamics:

Two thin metal discs of different heat capacities, are initially at temperatures T_1 and T_2 respectively, with $T_2 > T_1$. In this problem you may assume that the temperatures of the discs always remain uniform. The discs are then brought into thermal contact on their flat surfaces. The discs come to equilibrium under atmospheric pressure, in thermal isolation.

a) What is the relevant heat capacity for this situation, i.e, C_P or C_V? Assume that the heat capacities are constant with respect to temperature.

b) Find the final temperature.

c) Find the increase in entropy of the universe.

3. Electricity and Magnetism:

A simple pendulum of length l has at its end a mass m with total charge q. The pendulum is set in motion with small amplitude. Considering the radiation emitted by the charge, calculate how long it takes for the system to lose half of its energy.

4. Mechanics:

Two people are holding the ends of a uniform rod of length l and mass m. One person suddenly releases their end. Find the linear acceleration of the free end of the rod at the instant of release.

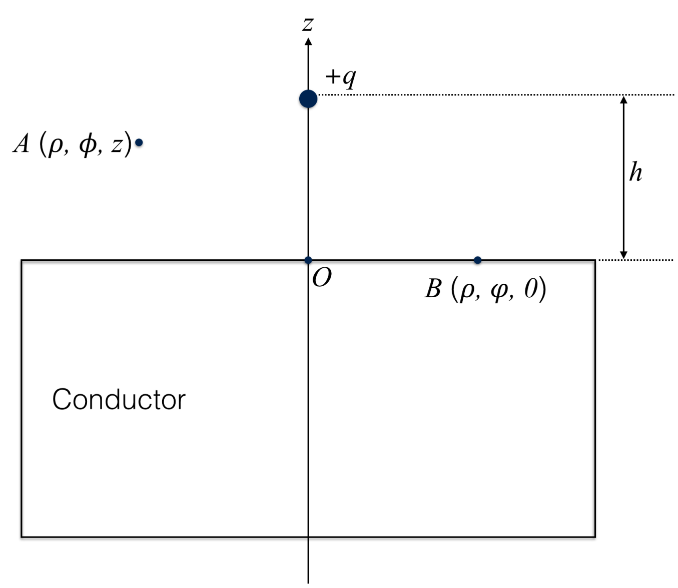
5. Electricity and Magnetism:
A point charge $+q$ is located a distance h above the surface of an infinite, flat conducting plane that is grounded as shown in the figure below [Hint: it is more convenient to use cylindrical coordinates].

a) Calculate the electric potential $\Phi(\mathbf{r})$ outside the conductor; i.e., calculate the electric potential at point A.

b) Calculate the electric field $\mathbf{E}(\mathbf{r})$ on the surface of the conducting plane; i.e., calculate the electric field at point B.

c) Calculate the surface charge density σ on the conductor.

d) Calculate the net induced charge $q_{induced}$ on the conductor.



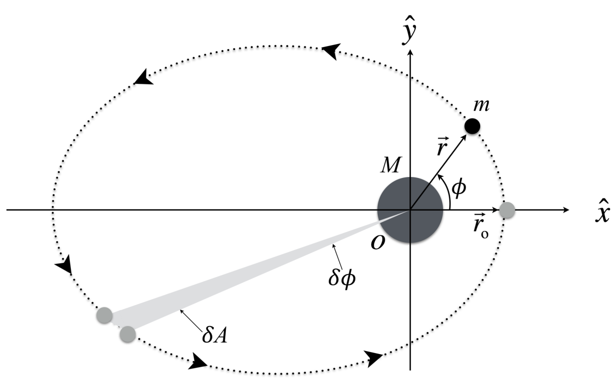
6. Electricty and Magnetism:

A cylindrical capacitor stands vertically with one end in a tank of dielectric oil of susceptibility χ and density ρ. The inner conductor of radius a is held at voltage V and the outer conductor of radius b is held at ground. Find the height h to which the oil rises in the space between the tubes.

7. Mechanics:

Every planet moves in an ellipse with the sun at one focus. Consider a planet of mass m orbiting the sun of mass M under the gravitational interaction $\overrightarrow{F}(r) = -\overrightarrow{r}GMmr^{-3}$, where G is the gravitational constant and $\overrightarrow{r}$ is the position vector from the sun to the planet as shown in the figure below. For simplicity, let us consider the planet and the sun as an isolated system, and solve in the limit $m << M$. Note that this is a two-dimensional problem, and the most convenient generalized coordinates are r and ϕ.

a) Derive the Euler-Lagrange equations for this system in these coordinates.
b) Show that a line joining the planet and the sun sweeps out equal areas during equal intervals of time, i.e., Kepler's Second Law: $dA/dt = constant$. Note that the area of the small triangle swept out by the planet as shown in the figure below is $\delta A = \frac{1}{2}r^2\delta\phi$.
c) Re-write the radial equation of motion in terms of the constant angular momentum ℓ_o.
d) Write down the effective force $f_e(r)$, i.e., the net force $[m\ddot{r} = f_e(r)]$, for the radial equation of motion, and the corresponding effective potential $U_e(r)$.
e) Determine the minimum distance r_o of the planet from the sun in terms of G, M, m, and ℓ_0.



8. Optics:

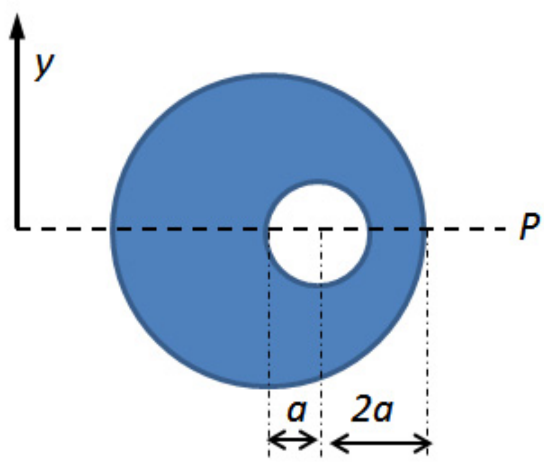
A soap bubble produces constructive interference in the reflected light when illuminated by light whose wavelength in air is 600 nm. The index of refraction of the soap bubble is 1.33. What is the minimum thickness of the bubble?

9. Electricity and Magnetism (Optional):

The figure shows the cross section of an infinitely long circular cylinder of radius $3a$ with an infinitely long cylindrical hole of radius a displaced so that its center is at a distance a from the center of the big cylinder. The solid part of the cylinder carries a current I, distributed uniformly over the cross section and out from the plane of the paper.

a) Find the magnetic field at all points on the plane P containing the axes of the cylinder.

b) Determine the magnetic field throughout the hole.



10. Optics (Optional):
A person has a far point of infinity and a near point of 83 cm. What is the refractive power of contact lenses this person should wear in order to be able to read a cell phone screen located 25 cm from her eyes?

11. Mechanics (Optional):

At the top of the Dennison building at height $h = 50$ m at latitude $\lambda = 42^0$ on the earth, which rotates with angular velocity $\omega = \dfrac{2\pi}{\text{day}}$, a marble is dropped down an elevator shaft. Find the direction and magnitude of the deflection from a perfectly vertical fall when the marble strikes the bottom of the shaft.

12. Thermodynamics (Optional):
The emission of radiation from the sun can be modeled by black body radiation using a temperature of 5800 K. The diameter of sun subtends an angle of 0.53 degrees at the earth's surface.

a) Assuming that the earth absorbs and re-emits as a black body the electromagnetic radiation incident upon it from the sun, calculate the surface temperature of the earth at steady state.

b) Calculate the power per unit area that is incident on earth. If you have trouble with (a), solve this in terms of T_E.