Name: ______________________________

University of Michigan Physics Department Graduate Qualifying Examination

Part II - Modern Physics

Saturday, May 16, 2009 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$\text{speed of light} \quad c = 2.998 \times 10^8 \text{ m/s}$$
$$\text{electron charge} \quad e = 1.602 \times 10^{-19} \text{ C}$$
$$\text{Planck's constant} \quad h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$$
$$\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s}$$
$$\text{Rydberg constant} \quad R_\infty = 1.097 \times 10^7 \text{ m}^{-1}$$
$$\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$
$$\text{Universal gas constant} \quad R = 8.3 \text{ J/K} \cdot \text{mol}$$
$$\text{Avogadro's number} \quad N_A = 6 \times 10^{23} \text{ mol}^{-1}$$
$$\text{Boltzmann's constant} \quad k_B = R/N_A = 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5} \text{eV/K}$$
$$\text{Stefan -- Boltzmann constant} \quad \sigma = 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4$$
$$\text{radius of the sun} \quad R_{sun} = 6.96 \times 10^8 \text{ m}$$
$$\text{radius of the moon} \quad R_{moon} = 1.74 \times 10^6 \text{ m}$$
$$\text{radius of the earth} \quad R_{earth} = 6.37 \times 10^6 \text{ m}$$
$$\text{Gravitational constant} \quad G_N = 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}$$

PART A: Obligatory Problems

1. (Quantum Mechanics) The Hilbert space of a quantum system consists of two orthonormal eigenstates $|a_1\rangle$ and $|a_2\rangle$ of a hermitian operator A with eigenvalues of +1 and -1, respectively.

   $A|a_1\rangle = +|a_1\rangle$ and $A|a_2\rangle = -|a_2\rangle$

   Another operator B is defined by:

   $B|a_1\rangle = i|a_2\rangle$ and $B|a_2\rangle = -i|a_1\rangle$ where i = $\sqrt{-1}$

   a) Express $|b_1\rangle$ and $|b_2\rangle$, the orthonormal eigenstates of B, in terms of $|a_1\rangle$ and $|a_2\rangle$.

   b) What are the corresponding eigenvalues of B ?

   c) What are the expectation values $\langle B^2\rangle$ and $\langle B\rangle$ for the state $|a_1\rangle$?

2. (Quantum Mechanics) A particle of mass m is in a potential

$$V(x) = -\alpha(\delta(x-a) + \delta(x+a)), \tag{1}$$

with $\alpha > 0$.

a) Determine the wave function for a bound state with odd parity? You do not need to fix the overall normalization of the wave function.

b) By imposing the appropriate boundary conditions, determine the condition on α that ensures the existence of such an odd-parity bound state.

3. (Quantum Mechanics) A quantum particle has escaped a potential, so that it has positive energy E. Assume that it is traveling in the $+x$ direction. At large x, the potential has the form

$$V(x) = -V_0 \frac{\ell}{x} \, ,$$

where the length scale ℓ is the location where $E + V(x) = 0$. Use the WKB method to find an approximate solution to the Schrödinger Wave Equation for large $x \gg \ell$.

4. (Quantum Mechanics) Consider the axially symmetric rotor with Hamiltonian

$$H = \frac{L_x^2 + L_y^2}{2I_1} + \frac{L_z^2}{2I_2}. \tag{2}$$

a) Solve for the energy spectrum of this system.
b) Now suppose an external magnetic field is applied: $B_{ext} = B_x\hat{\imath} + B_y\hat{\jmath} + B_z\hat{k}$. The magnetic dipole moment associated with the motion of the rotor is given by $\mu = -c\,\mathbf{L}$, with c a constant. To leading order in the magnetic field, compute the correction to the energies of the rotor.

5. (Statistical Physics) Consider a simple physical system with an infinite number of energy levels $E_n = nE_0$ where n is any integer $n \geq 0$ and where E_0 is a (constant) energy scale. The system is in contact with a reservoir of temperature T. Next assume that the degeneracy of each energy level is unity, i.e., there exists one and only one energy level for each value of n. Derive expressions for the partition function Z and the mean energy $\langle E \rangle$ as a function of temperature. Evaluate the energy (find simpler expressions correct to leading order) in both the high temperature $T \to \infty$ and low temperature $T \to 0$ limits.

6. (Statistical Physics) Suppose you leave a glass bulb of oxygen (O_2) at 1 atmosphere of pressure in a large box of helium (He), also at 1 atmosphere. This type of glass is permeable to He but not to O_2. After a long time what is the pressure inside the bulb? Explain.

7. (Statistical Physics) A gas of atoms at temperature, T, with mass, m, emits light. If the atoms of gas were at rest, they would all emit at frequency ν_o. However, the Doppler shift means that the frequency emitted depends on v_x, where x is the line-of-sight to the spectroscope. In fact, we know from the theory of the Doppler shift:

$$\nu = \nu_o[1 + v_x/c]$$

where c is the velocity of light.

a.) Find the mean frequency emitted, $\langle \nu \rangle$.

b.) Find $\Delta\nu = \sqrt{\langle(\nu - \nu_o)^2\rangle}$, the Doppler-broadened linewidth.

8. (Nuclear Physics) Archaeologists uncover an ancient grave containing an axe handle made of oak wood. A sample from this handle yields 19.5 carbon-14 decays per minute. A sample of the same mass taken from a nearby living oak tree yields 32.7 carbon-14 decays per minute. Carbon-14 has a half-life of 5730 years.

   a) How old (in years) is this ancient axe handle?

   b) What is the origin of carbon-14 in living organisms? How is this carbon-14 produced?

PART B: Optional Problems

9. (Atomic Physics) Application of the Born approximation. A repulsive central potential has the form $V(r) = A/r^2$. Find the differential scattering cross section of the potential $V(r)$ for a particle of mass m in the Born approximation. Express your result as a function of the scattering angle, θ, and the magnitude of the wave-vector, k.

   $\int_0^\infty \frac{\sin ax}{x} dx = \frac{\pi}{2}$ for positive a.

10. (Atomic Physics) In moderate magnetic fields, the sodium atom follows the Russell-Saunders (or L-S) coupling scheme. The atom has a ground state $3\ {}^2S_{1/2}$ and an excited state $3\ {}^2P_{3/2}$. We ignore hyperfine effects. The wavelength of a laser beam driving the transition between these states is $\lambda = 588.995$ nm (aka the D2 line).

    (a) What are the values of the intrinsic electron spin S, the orbital electron angular momentum L, and the combined angular momentum J for both the ground and excited state?

    (b) A magnetic field of $B = 6$ Tesla is applied. Draw a qualitative level diagram that shows the Zeeman splitting of both the ground and the excited state, with the correct number of Zeeman-split magnetic sub-levels. Assign proper values of the magnetic quantum number to the sub-levels in your drawing.

    (c) Determine the Lande g-factors of the ground and excited level.

    (d) What is the wavelength of the transition $3\ {}^2S_{1/2}\ |m_J = -1/2\rangle \rightarrow 3\ {}^2P_{3/2}\ |m_J = -1/2\rangle$ in the magnetic field? (Give 6 significant digits)

    The Bohr magneton is $\mu_B = 9.2740 \times 10^{-24}$ Am^2.

11. (Particle Physics) An experiment is underway to look for the rare decay $\mu \to e\gamma$.
a) What is the energy of the electron in this decay, if the muon is at rest?
b) Why is this decay rare? What factors suppress this decay relative to the dominant decay mode for the muon?

12. (Condensed Matter Physics) Describe an experimental technique which can be used to measure the band gap of a direct band gap semiconductor. In a direct band gap semiconductor the conduction band minimum and the valance band maximum have the same crystal momentum.