Name: ______________________________

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics

Saturday, May 9, 2009 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$\begin{aligned}
\text{speed of light} \quad c &= 2.998 \times 10^8 \text{ m/s} \\
\text{electron charge} \quad e &= 1.602 \times 10^{-19} \text{ C} \\
\text{Planck's constant} \quad h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s} \\
\hbar = h/2\pi &= 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s} \\
\text{Rydberg constant} \quad R_\infty &= 1.097 \times 10^7 \text{ m}^{-1} \\
\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} &= 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \\
\text{Universal gas constant} \quad R &= 8.3 \text{ J/K} \cdot \text{mol} \\
\text{Avogadro's number} \quad N_A &= 6 \times 10^{23} \text{ mol}^{-1} \\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5} \text{eV/K} \\
\text{Stefan} - \text{Boltzmann constant} \quad \sigma &= 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4 \\
\text{radius of the sun} \quad R_{sun} &= 6.96 \times 10^8 \text{ m} \\
\text{radius of the moon} \quad R_{moon} &= 1.74 \times 10^6 \text{ m} \\
\text{radius of the earth} \quad R_{earth} &= 6.37 \times 10^6 \text{ m} \\
\text{Gravitational constant} \quad G_N &= 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}
\end{aligned}$$

PART A: Obligatory Problems

1. (Mechanics) Let the y-axis point vertically up and the x-axis point horizontally. A particle of mass m is constrained to move on a frictionless parabolic track described by the equation y=bx^2, where b is a constant. The only forces acting on the particle are gravity (mg downward) and the force of constraint exerted by the track. Let x be the generalized coordinate.

   a) What is the Lagrangian of the particle?

   b) Using Lagrange's equation, find the equation of motion for x(t).

   c) What is the period of small oscillations for this system?

   d) What is the condition on the amplitude of oscillations for the oscillations to be approximately harmonic?

2. (Mechanics) Consider a damped and forced oscillator, where the oscillatory behavior is governed by the differential equation

$$\frac{d^2x}{dt^2} + 2\gamma\frac{dx}{dt} + \omega^2 x = F(t)\,,$$

where γ is the damping coefficient, ω is the free oscillation frequency, $F(t)$ is the forcing function. We also consider sufficiently weak damping so that $\gamma < \omega$. Consider the case where the oscillator is at rest for $t < 0$, and it experiences a sharp impulse at $t = 0$. Specifically, we let $F(t) = Q\delta(t)$, where δ is the Dirac delta function. Find the solution $x(t)$ for the subsequent motion.

3. (Mechanics) Consider two masses M_1 and M_2 orbiting each other in a circular orbit with period P. Assume that at a particular instant of time, the two masses experience a large instantaneous acceleration so that the direction of their velocity changes from exactly circular motion to exactly radial (inward) motion with no change in speed. In other words, they are turned so that they start to fall in towards each other. Find the time required for the two masses to collide.

4. (E&M) A stationary sphere with radius 5 cm carries a charge of 1 μC that is evenly spread out through the volume of the sphere. A test charge of -1 nC and mass 0.01g is released from rest at the surface of the sphere, at location $(0, 0, 5 \text{ cm})$, at time $t = 0$. The setup is such that the test charge can move in a friction-less manner (both outside and inside the sphere). Find the position vector of the test charge as a function of time. Ignore gravity. Provide number values in your final answer.

5. (E&M) The LHC CMS experiment uses one of the world's largest superconducting solenoidal magnets. The 21.6 m long solenoid has a diameter of 6.0 m and an inductance of 14.2 henry. The maximum current is 19,500 amperes and the total solenoidal circuit resistance is 0.00010 ohm.

   a) What is the stored energy (in joules) at the maximum current?

   b) If the magnet power supply is turned off, how much time (in seconds) is required for the current to drop from its maximum value to 1000 amperes?

   c) Compute the approximate number of conductor windings of the solenoid.

   d) Compute the approximate, average magnetic field (in tesla) in the solenoid at maximum current.

6. (E&M) Two unequal capacitors are charged separately to the same potential difference V, and subsequently the positive terminal of one is connected to the negative terminal of the other. Then the two outermost connections are shorted together.

   (a) Calculate the final charge on each capacitor.

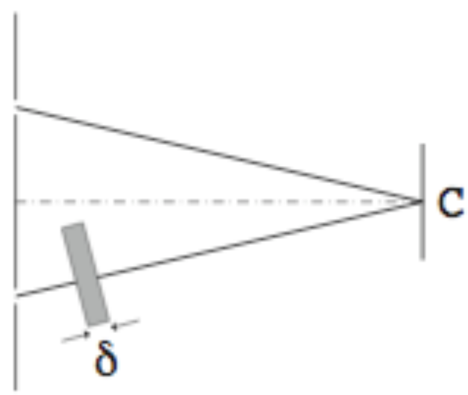
   (b) Calculate the loss in electrostatic energy.

7. (Optics) The figure below shows a double slit experiment in which monochromatic light of wavelength λ from a distant source is incident upon two slits, each of width w ($w << \lambda$), and the interference pattern is viewed on a distant screen. A thin piece of glass of thickness δ and index of refraction n, is placed between one of the slits and the screen. and the intensity at the central point C is measured as a function of δ. The intensity for $\delta = 0$ is given by I_0. (Assume that the glass does not absorb any light)

   (a) What is the intensity at point C as a function of thickness δ?

   (b) For what values of δ is the intensity at C a minimum?

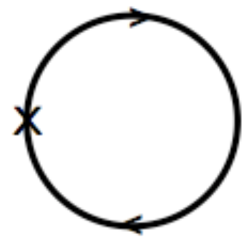
   (c) Suppose that the width of one of the slits is now increased to $2w$, the other width remaining unchanged. What is now the intensity at point C as a function of δ?



8. (Thermodynamics) He atoms can be viewed as hard spheres with a diameter of 0.22 nm. Calculate at room temperature (300 K) the mean free path and scattering time of pure He gas at a pressure of 1 Atm (1.013×10^5 Pa). [Note: The mass of a He atom is $M_{He} = 6.7 \times 10^{-27}$ kg.]

PART B: Optional Problems

9. (Mechanics) A particle has a circular orbit as shown in the figure, with the force center at the location X. Determine the force law.



10. (E&M) Consider a long, grounded, hollow, conducting cylinder with an inner radius of $a = 10$ cm that is concentric with the z-axis. The cylinder extends to infinity along the $+z$-direction and is terminated at $z = 0$ by a planar bottom plate extending in the xy-plane. The bottom plate is held at a constant potential of $V_0 = 100$ V.

    (a) Write down a series solution for the potential inside the cylinder. Provide integral expressions for the coefficients in the series (don't evaluate integrals).

    (b) Make an approximation that is suitable for $z \gg a$. Evaluate the remaining expansion coefficient and provide a numerical result.

    (c) At what distance z does the electric field on the z-axis drop below 1 V/m?

Hint for a):

$\int_0^a \rho J_m(x_{mn'}\frac{\rho}{a}) J_m(x_{mn}\frac{\rho}{a}) d\rho = \frac{a^2}{2}\left[J_{m+1}(x_{mn})\right]^2 \delta_{n,n'}$

Hints for b) and c):

The first zero of J_0 is $x_{01} = 2.405$.

$J_1(x_{01}) = 0.5191$

$\int_0^a \rho J_0(x_{01}\frac{\rho}{a}) d\rho = 0.2159a^2$

$J_0(0) = 1$

11. (Optics and Mechanics) The University of Michigan is planning to launch a satellite for a physics experiment. The satellite will have a geostationary orbit and will send signals to Ann Arbor at a frequency of 15 GHz. Estimate the size of the parabolic antenna needed to direct the signals approximately to a circular region of radius 300 km around the Physics Department. [Note: The mass of earth is $M_E = 5.97 \times 10^{24}$ kg.]

12. (Thermodynamics) It is well known that the time it takes to cook a turkey is dependent on its size. Some cooking books recommend a baking time which is proportional to the weight of turkey (approximately 15 minutes per pound).

   (a) Assuming a uniform spherical turkey of radius r_t, write down a differential equation describing the temperature profile within the turkey during baking. Using this equation determine, how the cooking time would scale with r_t? You can assume all the relevant material parameters of turkey is independent of temperature.

   (b) Discuss how the cooking of a turkey should scale with its weight in this approximation?