Name: ______________________________

University of Michigan Physics Department Graduate Qualifying Examination

Part II - Modern Physics

Saturday, May 10, 2008 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

$$\begin{aligned}
\text{speed of light} \quad c &= 2.998 \times 10^8 \text{ m/s} \\
\text{electron charge} \quad e &= 1.602 \times 10^{-19} \text{ C} \\
\text{Planck's constant} \quad h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s} \\
\hbar = h/2\pi &= 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s} \\
\text{Rydberg constant} \quad R_\infty &= 1.097 \times 10^7 \text{ m}^{-1} \\
\text{Coulomb constant} \quad k = (4\pi\epsilon_0)^{-1} &= 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2 \\
\text{Universal gas constant} \quad R &= 8.3 \text{ J/K} \cdot \text{mol} \\
\text{Avogadro's number} \quad N_A &= 6 \times 10^{23} \text{ mol}^{-1} \\
\text{Boltzmann's constant} \quad k_B = R/N_A &= 1.38 \times 10^{-23} J/K = 8.617 \times 10^{-5} \text{eV/K} \\
\text{Stefan} - \text{Boltzmann constant} \quad \sigma &= 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4 \\
\text{radius of the sun} \quad R_{sun} &= 6.96 \times 10^8 \text{ m} \\
\text{radius of the moon} \quad R_{moon} &= 1.74 \times 10^6 \text{ m} \\
\text{radius of the earth} \quad R_{earth} &= 6.37 \times 10^6 \text{ m} \\
\text{Gravitational constant} \quad G_N &= 6.67 \times 10^{-11} m^3/kg/s^2 = 6.71 \times 10^{-39} GeV^{-2}
\end{aligned}$$

PART A: Obligatory Problems

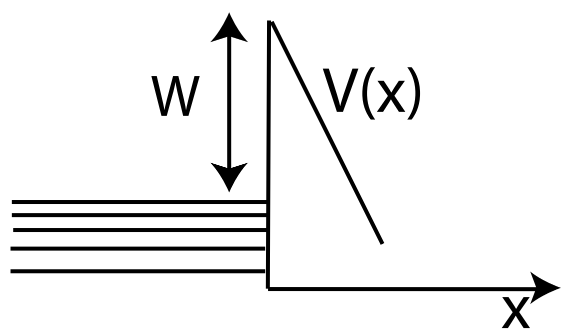
1. (Quantum Mechanics) Consider an electron (in one spatial dimension) that lives in an extremely deep and narrow potential well, so that you can model the potential as $V(x) = -Q\delta(x)$, where Q is a constant and $\delta(x)$ is the Dirac delta function. Find the energy eigenvalues of the system. How many bound states are allowed?

2. (Quantum Mechanics) An infinitely deep, one-dimensional (coordinate x) square well extends over a range $0 < x < a$ and contains a quantum particle of mass m. Initially, the particle is in the ground state.

   (a) The range of the well suddenly increases to $0 < x < b$ with $b > a$. Find the probability that the particle remains in the ground state.

   (b) If, instead, the range of the well increases very slowly to $0 < x < b$, over a time that is much longer than the inverse frequency separations of the relevant energy levels in the well, find the probability that the particle remains in the ground state.

3. (Quantum Mechanics) One can coax electrons from a metal by applying an electric field, E, to it. In the presence of the electric field, the potential can be approximated as shown. The depth of the potential well down to the Fermi surface is related to the work function necessary to eject electrons from the metal. Call this depth W. Using the WKB approximation, compute the transmission coefficient for electrons at the Fermi surface to tunnel out of the metal. Write your answer in terms of W, E, and fundamental constants.



4. (Quantum Mechanics) One can model the encounter between a free neutron and a nucleus as a sudden drop in potential energy from V=0 (outside) to V= -10 MeV inside. As a crude model consider the 1d problem:

$$V(x) = \begin{matrix} 0 & (-\infty < x < 0) \\ -10\text{ MeV} & (0 \geq x < \infty) \end{matrix} \tag{1}$$

Suppose a neutron emitted from a fission event has a kinetic energy of 3 MeV. When it encounters the next nucleus, estimate the probability that it gets absorbed, potentially causing another fission event, by computing the transmission probability for the neutron.

5. (Statistical Mechanics)

(a) A classical particle in an infinite parabolic potential well $V(x) \propto x^2$ oscillates back and forth around the origin with angular frequency ω. What are the energy levels of a spinless quantum particle obeying the Schrödinger equation in the same well?

(b) Find a closed-form expression for the partition function Z of such a particle in equilibrium at temperature T.

(c) If ω is the frequency of 100μm infrared and T is room temperature, how many levels will have greater than 1% probability of occupation?

6. (Statistical Mechanics) The density of states as a function of energy ϵ for a freely moving classical spinless particle of mass m in a box of volume V is

$$n(\epsilon) = \frac{V}{4\pi^2}\left(\frac{2m}{\hbar^2}\right)^{3/2}\epsilon^{1/2}.$$

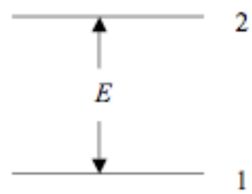
(a) Write down an integral representing the partition function Z_1 of such a particle and evaluate it using the result

$$\int_0^\infty \sqrt{x}e^{-ax}\,dx = \frac{\sqrt{\pi}}{2a^{3/2}}.$$

(b) A naive calculation of the partition function Z_N for N identical noninteracting such particles says that $Z_N = Z_1^N$. Using this formula and your expression for Z_1 find Z_N for a gas of spinless particles and hence find the entropy. Show that the resulting expression for the entropy is not extensive.

(c) What correction must be made to the expression for Z_N to make the entropy extensive?

7. (Statistical Mechanics) Consider an ensemble of systems that have only two energy levels with an energy spacing E. Sketch the average energy of a single system as a function of temperature, and on the same graph plot the specific heat as a function of temperature.



8. (Atomic Physics) A negative K-meson with mass $M = 1000m_e$ (m_e = electron mass) is captured into a tight circular Bohr orbit with principal quantum number $n_i = 12$ around a lead nucleus ($Z = 82$). Assume that the nucleus is initially fully stripped. The K-meson then cascades down through levels $n = 11, 10, 9, ...$ etc.

   (a) What is the energy in electron-Volts of the photon emitted in the $n_i = 12$ to $n = 11$ transition?

   (b) Experimentally, there are no quanta observed that are consistent with transitions into final states with quantum numbers less than $n = 3$ (because of fast absorption of the K-meson inside the nucleus). What is the approximate radius of the lead nucleus?

PART B: Optional Problems

9. (Atomic Physics) A tritium atom (^{3}H) in the electronic ground state decays into a Helium ion (^{3}He$^+$). What is the probability that the ion emerges in the electronic ground state? Note $\int_0^\infty x^n \exp(-px)dx = n!/p^{n+1}$

10. (Condensed Matter Physics) Calculate the density of states of a two-dimensional free electron system.

11. (Nuclear Physics)

(a) The nucleus ^{238}U (Z=92) spontaneously fissions, but with a long half life. Using the nuclear mass tables provided, estimate the energy (in MeV) released when this nucleus fissions.

(b) In what form does the energy that is initially released appear? (Provide as many details as you can.)

(c) Surprisingly ^{238}U does not fission symmetrically, but instead the fission products have different masses, m_1 and m_2. Why is this?

12. (Particle Physics) Neglecting CP violation and decay, the neutral B-Meson system ($B^0 = \bar{b}d, \bar{B}^0 = b\bar{d}$) can be described in this basis in terms of a two by two Hamiltonian of the form:

$$H = \begin{pmatrix} M & A \\ A & M \end{pmatrix} \tag{2}$$

a) In terms of the mass difference Δ_m between the two mass eigenstates, compute the probability for a B^0 meson produced at $t = 0$ to be measured as a B^0 meson at a later time t_0.

b) In one sentence, state how would you modify the above Hamiltonian to account for decays of the mesons?

c) What Standard Model force provides the off-diagonal piece to the above hamiltonian A?