

Name: _____

University of Michigan Physics Department Graduate Qualifying Examination

Part I - Classical Physics
Saturday, May 3, 2008 9:00am-1:00pm

This is a closed book exam - but you may use the materials provided at the exam. If you need to make an assumption or estimate, indicate it clearly. Show your work in an organized manner to receive partial credit for it.

You must answer the first 8 obligatory questions and two of the optional four questions. Indicate which of the latter you wish us to grade (e.g., circle the question number). We will only grade the indicated optional questions. Good Luck.

SOME FUNDAMENTAL CONSTANTS IN CONVENIENT UNITS

speed of light	$c = 2.998 \times 10^8 \text{ m/s}$
electron charge	$e = 1.602 \times 10^{-19} \text{ C}$
Planck's constant	$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} = 4.136 \times 10^{-15} \text{ eV} \cdot \text{s}$
	$\hbar = h/2\pi = 1.055 \times 10^{-34} \text{ J} \cdot \text{s} = 0.658 \times 10^{-15} \text{ eV} \cdot \text{s}$
Rydberg constant	$R_\infty = 1.097 \times 10^7 \text{ m}^{-1}$
Coulomb constant	$k = (4\pi\epsilon_0)^{-1} = 8.988 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
Universal gas constant	$R = 8.3 \text{ J/K} \cdot \text{mol}$
Avogadro's number	$N_A = 6 \times 10^{23} \text{ mol}^{-1}$
Boltzmann's constant	$k_B = R/N_A = 1.38 \times 10^{-23} \text{ J/K} = 8.617 \times 10^{-5} \text{ eV/K}$
Stefan – Boltzmann constant	$\sigma = 5.6703 \times 10^{-8} \text{ W/m}^2\text{K}^4$
radius of the sun	$R_{sun} = 6.96 \times 10^8 \text{ m}$
radius of the moon	$R_{moon} = 1.74 \times 10^6 \text{ m}$
radius of the earth	$R_{earth} = 6.37 \times 10^6 \text{ m}$
Gravitational constant	$G_N = 6.67 \times 10^{-11} \text{ m}^3/\text{kg/s}^2 = 6.71 \times 10^{-39} \text{ GeV}^{-2}$

PART A: Obligatory Problems

1. (Mechanics) The earth today can be approximated as a uniform sphere with a density of 5.5 gr/cm^3 . What was the length of a day when the earth was cooling and had a density that was half of the present density? Discuss qualitatively how your result would differ for a hollow earth where all mass is concentrated in a thin spherical shell.

2. (Mechanics) Consider three equal masses (with mass m) coupled together by four springs with spring constant k , all confined to move along a single line as shown below.
- (a) Find the Lagrangian and the equations of motion of the system.
- (b) Find the normal modes (eigenfrequencies) of the system.



3. (Mechanics) A small hamster is put into a circular wheel-cage, which has a frictionless central pivot. A horizontal platform is fixed to the wheel below the pivot. Initially, the hamster is at rest at one end of the platform.



When the platform is released the hamster starts running, but because of the hamster's motion, the platform and wheel remain *stationary*. Determine how the hamster moves.

4. (E&M) An infinite sheet is placed on the xy -plane and it is charged such that the potential on its surface is $V(x, y, 0) = V_0 \sin(kx)$.

(a) Calculate the electric potential, $V(x, y, z)$, below and above the sheet.

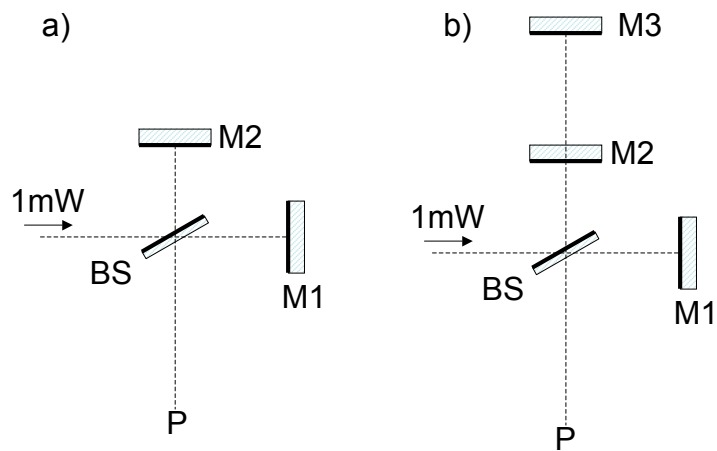
(b) Determine the surface charge density $\sigma(x, y)$ of the sheet.

5. (E&M) A parallel plate capacitor made of circular disks of area A separated by a distance d are connected by long straight wires to a battery with potential difference V across its terminals and in series with a resistance R .
- (a) If the connection is made at time, $t = 0$, find the electric field $E(t)$ inside the capacitor (you may assume that $A \gg d^2$ and calculate the field far from the edges).
 - (b) Is there a magnetic field inside the capacitor? If so, calculate it and its time dependence.

6. (E&M) An electromagnetic wave entering a conductor is damped exponentially with distance. The distance at which the amplitude of the wave is damped by a factor of $1/e$ is called the skin depth. Calculate for a conductor with a conductivity of σ the skin depth as a function of frequency. You should assume that σ is a constant and does not depend on frequency.

7. (Optics) A Michelson interferometer is constructed as in panel (a) below, with an $R = T = 50\%$ beamsplitter and two highly reflecting mirrors ($R = 99\%$). A well collimated, 1mW HeNe laser is directed into the perfectly aligned interferometer.

- What is the condition that complete destructive interference is seen at point P? Where did the incident light go?
- While in the condition of destructive interference, a second (parallel) mirror is added to one arm of the interferometer as shown in panel (b). The distance between the two mirrors in that arm is scanned. Plot the power seen at point P, as a function of this distance.

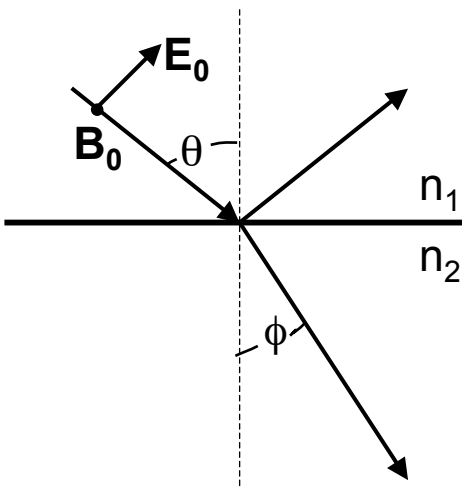


M1,M2,M3 Mirrors ($R=99\%$). BS 50/50% Beamsplitter

8. (Thermodynamics) 2.0 mol of He gas initially at 60 liters and 500 K undergoes an adiabatic expansion that doubles its volume. What is the temperature and pressure of the gas after the expansion? What is the work done by the gas during this expansion? (*Hint: In an adiabatic process there is no heat transfer into or out of a system.*)

PART B: Optional Problems

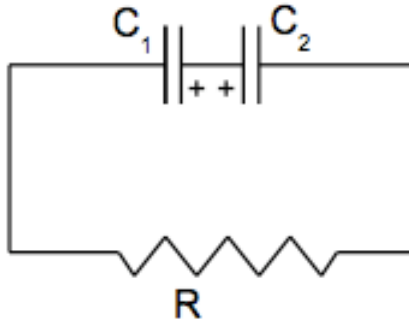
9. (Optics) Monochromatic plane waves are incident on a planar interface between two dielectric media of refractive index n_1 and n_2 . Derive an expression for the reflection coefficient (ratio of the reflected field to the incident field) as a function of incidence angle, θ , for transverse-magnetic (p-polarized) light. Show that there is no reflected light when $\tan(\theta) = n_2/n_1$.



10. (Mechanics) Consider the projectile motion of a Frisbee in the x - z plane. Specifically, the Frisbee is a disk shaped projectile of mass m , subject to a constant gravitational force, a drag force of magnitude $F_d = D\dot{x}^2$, and a lift force of magnitude $F_\ell = L\dot{x}$. The initial coordinates are taken to be $(x_0, z_0) = (0, h)$ and the initial velocity is $(\dot{x}_0, \dot{z}_0) = (v_0, 0)$. Find the equations of motion for the Frisbee and solve them.

11. (E&M) Two identical parallel plate capacitors, C_1 and C_2 , each with capacitance C are connected in series with a resistor R as shown below. Initially the charge stored on each capacitor is q_0 .

- (a) Describe (qualitatively and quantitatively) what happens in the circuit after the plate separation of capacitor C_1 is suddenly doubled at time $t = 0$. In particular, describe the charge on C_2 as a function of time t in terms of C , R and q_0 . What are the charges on C_1 and C_2 after a long time has passed?
- (b) Write down expressions for the total energy stored on the two capacitors before, immediately after, and a long time after the plate separation was changed. Explain any change quantitatively: where does any lost energy go, and what is the origin of any increase in energy?



12. (Thermodynamics) A 2kg piece of iron at 10°C is thrown into a large pool of water at 20°C . You can assume the pool is so large that its temperature is not cooled appreciably by the iron. The specific heat of iron at constant pressure between 10°C and 20°C can be treated as constant and equal to $450\text{ J kg}^{-1}\text{K}^{-1}$.
- (a) Treating the pool and the iron together as a thermally isolated system at constant pressure, calculate the increase in entropy of the combined system—both pool and iron, and not just the iron alone—as the iron warms up to 20°C .
 - (b) Show that the maximum amount of work that could have been extracted by running a heat engine between the water and the iron (instead of just throwing the iron into the pool) is 157 J.