

AIM Qualifying Review Exam: Probability and Discrete Mathematics

August 20, 2026

There are five (5) problems in this examination.

There should be sufficient room in this booklet for all your work. But if you use other sheets of paper, be sure to mark them clearly and staple them to the booklet.

Problem 1

Recall that a random variable X is *Poisson*-distributed with parameter $\lambda > 0$ if

$$\Pr(X = i) = e^{-\lambda} \frac{\lambda^i}{i!}$$

and that Y is *binomial* distributed with parameters n, p if $\Pr(Y = i) = \binom{n}{i} p^i (1 - p)^{n-i}$.

- (a) Given n, p , put $\lambda = np$. Show that, if X is λ -Poisson-distributed and Y is (n, p) -binomial-distributed, then, under conditions on i, n, p , $\Pr(X = i)$ is approximated by $\Pr(Y = i)$. State weak general assumptions on i, n, p , of the form i, n, p large or small compared with 1 or with each other, etc., that guarantee the approximation.
- (b) Suppose each math lesson contains an error, independently, with probability 0.1. What is the probability that ten lessons contain at most one error? Write an unsimplified expression using binomial symbols and show work.
- (c) Approximate the above probability by a suitable Poisson approximation. Write an unsimplified expression using exponential and show work.

Problem 1

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Problem 2

Let X be a random variable with probability density function

$$f(x) = \begin{cases} c(1 - x^2), & -1 < x < 1; \\ 0, & \text{otherwise.} \end{cases}$$

- (a) What is the value of c ?
- (b) What is the associated cumulative distribution function, $F(x)$?
- (c) Sketch f , and label minima, maxima, increase, decrease, and asymptotes.
- (d) Sketch F , and label minima, maxima, increase, decrease, and asymptotes.

Problem 2

Problem 2

Problem 2

Problem 3

How many permutations of the letters M, A, T, H, I, S, F, U, N are there in which none of the three words MATH, IS, FUN appear as consecutive substrings?

Show work, and give an unsimplified expression using factorial, binomial symbols, parentheses, $+$, $-$, $\times$, $\div$.

Problem 3

Problem 3

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Problem 4

Determine the generating function for the sequence of squares, i.e., find a closed form for $0^2 + 1^2x + 2^2x^2 + 3^2x^3 \dots$. You need not address radius of convergence.

Problem 4

Problem 4

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Problem 5

A Boolean circuit on TRUE/FALSE values is an arrangement of input gates, 2-input AND gates, 2-input OR gates, 1-input NOT gates, and a single output gate, organized and evaluated in the natural way. For example $x \vee \neg(y \wedge z)$. Here $\neg$ is not, $\wedge$ is AND, and $\vee$ is OR.

A circuit is satisfiable if there is some setting of TRUE/FALSE to the input gates that makes the output gate TRUE.

A 3-CNF (for Conjunctive Normal Form) is a giant AND of 3-wise ORs of boolean variables and their negations, often written using overlines for negation, e.g., $(x \vee \overline{y} \vee z) \wedge (\overline{u} \vee \overline{x} \vee \overline{z}) \wedge \cdots$. Each 3-wise OR is called a clause.

Show how, given a circuit ψ , to quickly build a 3-CNF φ , such that ψ is satisfiable iff φ is satisfiable.

Illustrate by giving **representatives** of the clauses that arise in $x \vee \neg(y \wedge z)$.

Problem 5

Problem 5

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