AIM Qualifying Review Exam in Advanced Calculus & Complex Variables

January 2026

There are five (5) problems in this examination.

There should be sufficient room in this booklet for all your work. If you use other sheets of paper, be sure to mark them clearly and staple them to the booklet. No credit will be given for answers without supporting work and/or reasoning.

Problem 1

Identify whether each of the following statements are true or false, and provide a proof or a counterexample as appropriate:

(a) A uniformly continuous function $f : (0, 1) \to \mathbb{R}$ is bounded.

(b) If $f : \mathbb{R} \to \mathbb{R}$ maps Cauchy sequences to Cauchy sequences, then f is continuous.

(c) The pointwise limit of a sequence of uniformly continuous functions on $\mathbb{R}$ is uniformly continuous.

(d) If $f, g : \mathbb{R} \to \mathbb{R}$ are uniformly continuous, then their product fg is uniformly continuous.

Problem 1

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Problem 2

Let

$$f(x) = \begin{cases} 1 + x & x > 0 \\ 0 & x = 0 \\ -1 + x & x < 0 \end{cases}$$

and consider the minimization problems

$$\text{Problem I:} \quad \min_{c \in \mathbb{R}} \int_{-1}^{1} |f(x) - c|^2 \, dx \qquad \text{and} \qquad \text{Problem II:} \quad \min_{c \in \mathbb{R}} \int_{-1}^{1} |f(x) - c| \, dx.$$

(a) Produce an optimal c for each problem. Be sure to justify your steps, e.g., if you decide to commute two limits as part of a calculation you must explain why that is possible. (Hint: it is not necessary to commute limits to solve this problem; if you are stuck, try drawing a graph.)

(b) One of these problems has more than one minimizer. Identify which problem and find all of its minimizers.

Problem 2

Problem 2

Problem 2

Problem 3

Evaluate

$$\int_{-\infty}^{\infty} \frac{x^2}{(1+x^2)(3+x^2)^2}\,dx$$

Problem 3

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Problem 4

Consider the complex polynomial

$$p(z) = 2z^4 + z^3 + 8z - 4.$$

How many zeros counting multiplicity does p have in the annulus $1 < |z| < 2$?

Problem 4

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Problem 5

Recall that the real and imaginary parts of a complex analytic function are harmonic. Using this, find a harmonic function $v = v(x, y)$ on the unit disc $x^2+y^2 < 1$ such that $v = 0$ for $x^2+y^2 = 1$ and $x > 0$, and $v = 1$ for $x^2 + y^2 = 1$ and $x < 0$. Your answer may involve the complex variable $z = x + iy$.

Problem 5

Problem 5

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