

AIM Qualifying Review Exam in Advanced Calculus & Complex Variables

August 2026

There are five (5) problems in this examination.

There should be sufficient room in this booklet for all your work. If you use other sheets of paper, be sure to mark them clearly and staple them to the booklet. No credit will be given for answers without supporting work and/or reasoning.

Problem 1

Assume f is a **strictly decreasing and continuous** function on $\mathbb{R}$ and $\{x_n\}$ is a bounded sequence.

Prove: $\liminf_{n \rightarrow \infty} f(x_n) = f(\limsup_{n \rightarrow \infty} x_n)$.

Problem 1

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Problem 2

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ be a **smooth** function. Prove the the following two statements are equivalent:

- For any $x, \xi \in \mathbb{R}^n$,

$$\sum_{i,j=1}^n \frac{\partial^2 f}{\partial x_i \partial x_j}(x) \xi_i \xi_j \geq 0.$$

- For any $x, y \in \mathbb{R}^n$,

$$(\nabla f(x) - \nabla f(y)) \cdot (x - y) \geq 0$$

Problem 2

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Problem 2

Problem 3

Let E be the solid region in $\mathbb{R}^3$ lying above the cone

$$z = \sqrt{x^2 + y^2}$$

and below the sphere

$$x^2 + y^2 + z^2 = 16.$$

- (a) Describe the region E using spherical coordinates $x = \rho \cos \theta \sin \phi$, $y = \rho \sin \theta \sin \phi$, $z = \rho \cos \phi$.
- (b) Evaluate the integral

$$\iiint_E z \, dV.$$

- (c) Rewrite the same integral in cylindrical coordinates $x = r \cos \theta$, $y = r \sin \theta$, $z = z$, including all limits of integration.

Problem 3

Problem 3

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Problem 4

Evaluate the real integral

$$\int_{-\infty}^{\infty} \frac{\cos(3x)}{(x^2 + 1)^2(x^2 + 4)} dx.$$

Problem 4

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Problem 5

1. Let F be an analytic function on $\mathbb{C}$ satisfying $|F(z)| \leq \sqrt{|z|}$. Prove $F(z) = 0$.

Hint: Write a bound for the first derivative of F .

2. Let U be a bounded and connected open set in $\mathbb{C}$ and let F be continuous on $\overline{U}$ and analytic on U . Assume F has no zeros in U and $|F(z)| = 1$ on the boundary of U . Prove that F is a constant function.

Hint: Consider $1/F(z)$ and open mapping theorem.

Problem 5

Problem 5

Problem 5