Department of Mathematics
Winter 2023 Graduate Course Descriptions

*****Math 498 | Topics in Modern Mathematics | DeBacker, S. | MR 10:00-11:30 PM**
Topic: Representation Theory of Compact and Finite Groups

The goal of this course is to provide an introduction to representation theory of finite and compact groups. Representation theory may be thought of as the study of symmetry in the context of vector spaces. Preparation for this course includes a thorough understanding of algebra (including vector spaces, groups, and group actions) and analysis (including the basics of measure theory). Students will complete weekly homework assignments and two group projects.

****MATH 501 | AIM Student Seminar | Alben, S. | Fri 1:00 PM – 2:00 PM**
Fri 3:00 PM – 4:00 PM

At least two 300 or above level math courses, and Graduate standing; Qualified undergraduates with permission of instructor only. (1). May be repeated for a maximum of 6 credits. Offered mandatory credit/no credit.

MATH 501 is an introductory and overview seminar course in the methods and applications of modern mathematics. The seminar has two key components: (1) participation in the Applied and Interdisciplinary Math Research Seminar; and (2) preparatory and post-seminar discussions based on these presentations. Topics vary by term.

No book for this course.

****MATH 506/IOE | Stochastic Analysis for Finance | Bayraktar, E. | TR 10:00 AM - 11:30 AM**

Math 526. Graduate students or permission of instructor. (3). (BS). May not be repeated for credit.

The aim of this course is to teach the probabilistic techniques and concepts from the theory of stochastic processes required to understand the widely used financial models. In particular concepts such as martingales, stochastic integration/calculus, which are essential in computing the prices of derivative contracts, will be discussed. The specific topics include: Brownian motion (Gaussian distributions and processes, equivalent definitions of Brownian motion, invariance principle and Monte Carlo, scaling and time inversion, properties of paths, Markov property and reflection principle, applications to pricing, hedging and risk management, Brownian martingales), martingales in continuous time, stochastic integration (including It\^{o}'s formula), stochastic differential equations (including Feynman-Kac formula), change of measure (including Girsanov's theorem and change of numeraire), and, time permitting, stochastic control (including Merton problem). Applications from various areas of Finance (including, pricing of derivatives, risk management, etc) are used to illustrate the theory.

****MATH 521 | Life Contingencies II | Natarajan, R. | TR 10:00 AM - 11:30 AM**

MATH 520 with a grade of C- or higher. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

This course extends the single decrement and single life ideas of MATH 520 to multi-decrement and multiple-life applications directly related to life insurance. The sequence 520-521 covers the Part 4A examination of the Casualty Actuarial Society and covers the syllabus of the Course 150 examination of the Society of Actuaries. Concepts and Calculation are emphasized over proof.

****MATH 524 | Loss Models II | Moore, K. | TR 1:00 PM - 2:30 PM**

STATS 426 and MATH 523. (Prerequisites enforced at registration.) (3). (BS). May not be repeated for credit.

Risk management is of major concern to all financial institutions, especially casualty insurance companies. This course is relevant for students in insurance and provides background for the professional examination in Short-Term Actuarial Modeling offered by the Society of Actuaries (Exam STAM). Students should have a basic knowledge of common probability distributions (Poisson, exponential, gamma, binomial, etc.) and have at least Junior standing.

Content: Frequentist and Bayesian estimation of probability distributions, model selection, credibility, simulation, and other topics in casualty insurance.

****MATH 525/STATS | Probability Theory | Husson, J. | TR 8:30 AM – 10:00 AM**
Vig, A. | TR 10:00 AM - 11:30 AM
TR 1:00 PM - 2:30 PM

MATH 451, MATH 425/STATS 425 would be helpful. (3). (BS). May not be repeated for credit.

This course is a thorough and fairly rigorous study of the mathematical theory of probability at an introductory graduate level. The emphasis will be on fundamental concepts and proofs of major results, but the usages of the theorems will be discussed through many examples. This is a core course sequence for the Applied and Interdisciplinary Mathematics graduate program. This course is the first half of the Math/Stats 525-526 sequence.

****MATH 526/STATS | Discrete State Stochastic Processes | Das, P. | TR 8:30 AM - 10:00 AM**
Kim, D. | TR 1:00 PM - 2:30 PM

MATH 525 or STATS 525 or EECS 501. (3). (BS). May not be repeated for credit.

This is a course on the theory and applications of stochastic processes, mostly on discrete state spaces. It is a second course in probability which should be of interest to students of mathematics and statistics as well as students from other disciplines in which stochastic processes have found significant applications.

The material is divided between discrete and continuous time processes. In both, a general theory is developed and detailed study is made of some special classes of processes and their applications. Some specific topics include generating functions; recurrent events and the renewal theorem; random walks; Markov chains; branching processes; limit theorems; Markov chains in continuous time with emphasis on birth and death processes and queueing theory; an introduction to Brownian motion; stationary processes and martingales.

Required Textbook: Durrett, Richard. (2016). **Essentials of Stochastic Processes**, 3rd Ed. Springer.

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MATH 547/BIOINF/STATS   Probabilistic Modeling in Bioinformatics   Rajapakse, I.   TR 2:30 PM -4:00 PM

MATH, Flexible, due to diverse backgrounds of intended audience. Basic probability (level of MATH/STATS 425), or molecular biology (level of BIOLOGY 427), or biochemistry (level of CHEM/BIOLCHEM 451), or basic programming skills desirable or permission. (3). (BS). May not be repeated for credit.

This course is open to graduate students and upper-level undergraduates in applied mathematics, bioinformatics, statistics, and engineering, who are interested in learning from data. Students with other backgrounds such as life sciences are also welcome, provided they have maturity in mathematics. The mathematical content in this course will be linear algebra, multilinear algebra, dynamical systems, and information theory. This content is required to understand some common algorithms in data science. I will start with a very basic introduction to data representation as vectors, matrices, and tensors. Then I will teach geometric methods for dimension reduction, also known as manifold learning (e.g. diffusion maps, t-distributed stochastic neighbor embedding (t-SNE), etc.), and topological data reduction (introduction to computational homology groups, etc.). I will bring an application-based approach to spectral graph theory, addressing the combinatorial meaning of eigenvalues and eigenvectors of their associated graph matrices and extensions to hypergraphs via tensors. I will also provide an introduction to the application of dynamical systems theory to data including dynamic mode decomposition and the Koopman operator. Real data examples will be given where possible and I will work with you write code implementing these algorithms to solve these problems. The methods discussed in this class are shown primarily for biological data, but are useful in handling data across many fields. A course features several guest lectures from industry and government.

****551   INTRODUCTION TO REAL ANALYSIS   Le, H.   TR 11:30 AM – 1:00 PM**

This course introduces the measure theory and other topics of real analysis for advanced math undergraduates, master's students, and AIM and non-math graduate students. The main focus will be on Lebesgue measure theory and integration theory. Tentative topics included: Lebesgue measure, measurable functions, Lebesgue integral, convergence theorems, metric spaces, topological spaces, Hilbert and Banach spaces. This course has some overlaps with MATH 597, but covers about 1/2 of the content and proceeds at a slower pace.

Textbook: **Measure, Integration & Real Analysis** by Sheldon Axler
Free online version of textbook available

MATH 555   Introduction to Complex Variables   Barhoumi, A.   MW 8:30 AM - 10:00 AM
MATH 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

This course is an introduction to the analysis of complex-valued functions of a complex variable with an eye towards applications in science and engineering. Concepts, calculations, and the ability to apply principles to problems are emphasized alongside rigorous proofs of the basic results in the subject.

Topics covered include differentiation and integration of complex-valued functions of a complex variable, series, conformal mappings, residues, and applications including evaluation of improper real integrals and fluid mechanics. Applications to approximation theory will be covered if time permits.

Required Textbook: **Complex Analysis with Applications**, by Richard A. Silverman ISBN 978-0486647623

****MATH 557   Applied Asymptotic Analysis   Miller, P.   TR 1:00 PM - 2:30 PM**
MATH 217, 419, or 420; MATH 451; and MATH 555 or 596. (3). (BS). May not be repeated for credit.

Course content: Asymptotic analysis is the quantitative study of approximations. The fundamental idea is that one tries to solve a problem in applied mathematics (say, a boundary-value problem for a partial or ordinary differential equation) by embedding it into a family of problems with a parameter. If the problem can be solved exactly for one special value of the parameter, then asymptotic analysis can be used to analyze how the solution changes as the parameter is tuned from the special value to a more physically reasonable one. The course will develop the general theory of so-called asymptotic expansions, which are a kind of series in the perturbation parameter that are extremely useful in practice, in a way that is mathematically completely rigorous, despite the strange fact that they frequently fail to converge at all! We will then study how to use asymptotic expansions to evaluate integrals that cannot be computed in closed form and that are also difficult to approximate numerically. Next, we will turn to differential equations and use asymptotic expansions to evaluate solutions near certain singular points and also to study the way that solutions depend on parameters. At the end of the course we will study how the differential equations of diverse physical phenomena can be reduced, with the help of asymptotic expansions, to certain universal model equations that show up again and again in applied mathematics. Specific applications to be addressed in the course as time permits include the small-viscosity theory of shock waves, the theory of quantum mechanics in the semiclassical limit, aspects of the theory of special functions, vibrations in nonlinear lattices, and surface water waves.

Prerequisites: This course assumes background in differential equations, some linear algebra, and advanced calculus or elementary real analysis. Even more important is technical skill in complex variables and analysis at the level of Math 555 or Math 596.

Required Textbook: P. D. Miller, **Applied Asymptotic Analysis**, Graduate Studies in Mathematics, volume 75, American Mathematical Society, Providence, RI, 2006. ISBN 0-8218-4078-9.

Grading: There will be about 5 substantial homework sets, but no exams. Depending on the number of students enrolled in the course, there may also be reading/research projects with some deliverable possibly including in-class presentations.

****MATH 558   Applied Nonlinear Dynamics   Krasny, R.   MW 11:30 AM – 1:00 PM**

MATH 451. (3). (BS). May not be repeated for credit.

Dynamical systems theory provides tools for understanding complex behaviour in a broad range of applications. This course is an introduction to the theory. We will study continuous systems (differential equations) and discrete systems (iterated maps). The aim is to provide a broad overview of the subject as well as an in-depth analysis of specific examples. The topics include: bifurcations (saddle-node, transcritical, pitchfork, subcritical, supercritical, Hopf), stable and unstable manifolds, dissipative systems, attractors, logistic map, period-doubling, Feigenbaum sequence, renormalization, chaos, Lyapunov exponent, fractals, Cantor set, Hausdorff dimension, Lorenz system, nonlinear oscillations, quasiperiodicity, Hamiltonian systems, integrability, resonance, KAM tori, homoclinic intersections, Melnikov's method.

Free online textbook available

****Math 564 Topics in Mathematical Biology Forger, D. TR 10:00 AM – 11:30 AM**
Topic: Physiological Modeling and Prediction with Differential Equations and Machine Learning

Physiological systems are typically modeled by differential equations representing the state of their components, for example, Hodgkin and Huxley's mathematical description of the electrical activity of neurons. Black box methods using machine learning have recently had remarkable success in predicting physiological state in some settings, for example, in scoring sleep from wearables. This course will explore the differences between these two approaches and new techniques using both mechanistic differential equation models and machine learning. Topics include backpropagation methods for learning in artificial neuronal networks and biophysical neuronal networks, methods for filtering physiological signals (e.g., autoencoders) to serve as inputs to physiological models, machine learning methods to solve differential equations and learn dynamics, and new methods to study noise in biological systems. Final projects and teamwork will allow students to apply these techniques to their own choice of systems to study.

No textbook required

****MATH 566 Combinatorial Theory Seelinger, George TR 11:30 AM - 1:00 PM**
MATH 465 group theory and abstract linear algebra. (3). (BS). May not be repeated for credit.

We will discuss applications of algebra to combinatorics and vice versa. Topics may include: graph eigenvalues, random walks, domino tilings, matrix tree theorem, electrical networks, Eulerian tours, permutations, partitions, Young diagrams, Young tableaux, Sperner's theorem, Gaussian coefficients, RSK correspondence, partially ordered sets, etc.

Optional Textbook**: Algebraic Combinatorics: Walks, Trees, Tableaux, and More**, R. P. Stanley, ISBN 3319771728

****MATH 567 Introduction to Coding Theory Gadish, N. TR 2:30 PM - 4:00 PM**
One of MATH 217, 419, 420. (3). (BS). May not be repeated for credit.

This course introduces information theory, covering the concepts of entropy, Shannon's theorem, and channel capacity. We will further discuss noiseless coding and data compression. Our main tool will be linear algebra; thus, we will review these tools and introduce the relevant abstract algebra, finite fields, and polynomials over finite fields. Basic examples of codes we cover include Golay, Hamming, BCH, Reed-Muller, and Reed-Solomon codes. We will further discuss linear codes and cyclic codes and give fundamental asymptotic bounds on coding efficiency.

Required Textbook: **Introduction to Coding and Information Theory**, by S. Roman, ISBN: 978-0387947044

****MATH 571 Numerical Linear Algebra Viswanath, D. TR 10:00 AM - 11:30 AM**
MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454 or permission from the instructor.. (3). (BS). May not be repeated for credit.

This class is about solving linear systems numerically, finding eigenvalues and singular values, and solving linear least squares problems. We will discuss condition numbers, numerical stability, QR factorization, Cholesky, SVD, and the QR algorithm as well as iterative methods (GMRES, Arnoldi, Conjugate Gradients, Lanczos). The following applications are included: KKT conditions, convergence of the perceptron, and back propagation networks. The homework assignments will use either Python or Matlab, with the choice left to the student.

Required Textbook: **Numerical Linear Algebra**, by Lloyd N. Trefethen and David Bau; ISBN-13: 978-0898713619

****MATH 572 Numerical Methods for Differential Equations Esedoglu, S. TR 11:30 AM – 1:00 PM**
MATH 214, 217, 417, 419, or 420; and one of MATH 450, 451, or 454. (3). (BS). May not be repeated for credit.

Course Description: Math 572 is an introduction to numerical methods for differential equations, focusing on finite differences. This is a core course for the Applied and Interdisciplinary Mathematics (AIM) graduate program, and should also appeal to graduate students from engineering and science departments, or anyone interested in scientific computing. It covers methods for ordinary and partial differential equations, including derivation of numerical schemes and systematic study of their accuracy, stability, and convergence. A solid background in advanced calculus and linear algebra, and proficiency in a computer language such as C, Fortran, or Matlab is a must.

Topics:
Finite differences, their derivation and truncation error. Two point boundary value problems, elliptic equations. Consistency, stability, and convergence. Efficient solution of resulting sparse linear systems (Jacobi, Gauss-Seidel, SOR, conjugate gradients, preconditioning). Multistep, Runge-Kutta methods for initial value problems. Absolute stability, stiff problems, and A-stability. Barrier theorems. Explicit and implicit finite difference schemes for parabolic equations. Stability and convergence analysis via the maximum principle, energy methods, and the Fourier transform. Operator splitting techniques, the alternating direction implicit method. Advection equation. Lax-Wendroff, upwind methods, the CFL condition. Hyperbolic systems, initial boundary value problems.

Required Textbook**: Finite Difference Methods for Ordinary and Partial Differential Equations: Steady-State and Time-Dependent Problems** by R.J. LeVeque, ISBN: 978-0-898716-29-0

****MATH 574 Financial Mathematics II Norgilas, D. TR 1:00 PM – 2:30 PM**
MATH 526 and MATH 573. (Prerequisites enforced at registration.) Although MATH 506 is not a prerequisite for MATH 574, it is strongly recommended that either these courses are taken in parallel, or MATH 506 precedes MATH 574. (3). (BS). May not be repeated for credit.

This is a continuation of Math 573. This course discusses Mathematical Theory of Continuous-time Finance. The course starts with the general Theory of Asset

Pricing and Hedging in continuous time and then proceeds to specific problems of Mathematical Modeling in Continuous-time Finance. These problems include pricing and hedging of (basic and exotic) Derivatives in Equity, Foreign Exchange, Fixed Income and Credit Risk markets. In addition, this course discusses Optimal Investment in Continuous time (Merton's problem), High-frequency Trading (Optimal Execution), and Risk Management (e.g. Credit Value Adjustment).

Required Texts: **Arbitrage Theory in Continuous Time**, by Tomas Björk, 3rd 978- 0199574742
Stochastic Calculus for Finance II: Continuous-Time Models, by Steven E. Shreve, (2004) Springer, ISBN: 978-0387401010

****MATH 575 Introduction to Theory of Numbers I Snowden, A. MWF 1:00 PM – 2:00 PM**
MATH 451 and 420 or permission of instructor (Some background in abstract algebra – basics of groups, rings, fields – will be helpful). (1 - 3). (BS). May not be repeated for credit.

This course will be an introduction to number theory. Basic topics to be covered include factorization, congruences, Gauss and Jacobi sums, classical reciprocity laws such as quadratic and cubic reciprocity and some basic algebraic number theory.

No formal prerequisites, but some familiarity with abstract algebra, including the theory of groups, rings and fields will be assumed.

No textbook required

***MATH 582 Introduction to Set Theory Chen, R. TR 1:00 PM - 2:30 PM**
MATH 412 or 451 or equivalent experience with abstract mathematics. (3). (BS). May not be repeated for credit.

An introduction to axiomatic set theory, the foundations of mathematics, and the study of the infinite. We will cover topics including: the algebra of sets, the Zermelo- Fraenkel axioms of set theory, constructions of number systems, countable and uncountable sets, cardinals, ordinals, and the Axiom of Choice.

No textbook required

****MATH 590 Introduction to Topology McCleerey, N. MWF 12:00 PM - 1:00 PM**
MATH 451. (3). (BS). May not be repeated for credit. Rackham credit requires additional work.

This course is an introduction to point-set topology. We will cover topological spaces, continuous functions and homeomorphisms, the separation axioms, the quotient and product topology, compactness, connectedness, and metric spaces. We may also explore some topics in algebraic topology, time permitting.

Required Textbook: **Topology**, by James Munkres, ISBN: 978-0134689517

MATH 592 Introduction to Algebraic Topology Perry, A. MWF 10:00 AM - 11:00 AM
Previous exposure to point-set topology and familiarity with abstract algebra will be assumed. MATH 591. (3). (BS). May not be repeated for credit.

This course covers the basics of algebraic topology, including fundamental groups, covering spaces, and (co)homology, as well as some of their applications.

Optional textbook: **Algebraic Topology**, Allen Hatcher, ISBN: 978-0521795401

MATH 594 Algebra II Prasanna, K. TR 10:00 AM – 11:30 AM

Math 593 or instructor approval.

Math 594 is one of the "alpha" courses for the Math PhD program. Topics to be covered include the theory of groups, fields and Galois theory. If time permits, we will also cover the theory of (finite) group representations. The class will be a mix of lecture and IBL. No textbook is required, notes will be provided.

****MATH 597 Analysis II Jonsson, M. MWF 11:00 AM - 12:00 PM**
MATH 395/451 and 590. (3). (BS). May not be repeated for credit.

This is a course in Real Analysis for the PhD program in Mathematics. Topics include abstract measures, Lebesgue measure on R and R^n, measurable functions, integration, the Fubini theorem, complex and signed measures, the Lebesgue-Radon-Nikodym theorem, maximal functions, differentiation of measures, and L^p spaces. If time permits, we will also treat a bit of Hilbert space theory and Fourier analysis.

Required Textbook: **Real Analysis: Modern Techniques and Their Applications**, Gerald B. Folland. ISBN: 978-047-1317-16 (Required)

****MATH 605 Complex Analysis Barrett, D. MWF 1:00 PM - 2:00 PM**
Prerequisites: first-year graduate analysis [complex and real]

Domains in the complex plane always have a rich enough collection of holomorphic functions to solve a number of natural problems; for example, any complex-valued function on a discrete subset of the domain extends to a holomorphic function on the whole domain.

In higher dimension the situation is different: the extension problem stated above is automatically solvable on some domains, such as the unit ball, but not on others, such as the punctured ball. This last statement follows from the astounding fact that all isolated singularities are removable in higher dimension. There are also more subtle examples where all holomorphic functions on one domain automatically extend to a fixed larger domain, a phenomenon completely absent in one variable.

Notions developed to sort out these and many related phenomena include the concepts of pseudoconvexity for sets and plurisubharmonicity for functions. Both of these notions are generalizations of the real-variable notion of convexity. These and related concepts from the theory of several complex variables have important applications in many other parts of mathematics, including algebraic geometry and symplectic geometry.

An important tool for building holomorphic functions of several variables is the theory of the inhomogeneous Cauchy-Riemann equations. The study of these equations has also had important consequences elsewhere in mathematics, such as Hans Lewy's surprising discovery of a linear partial differential equation with no local solutions.

Math 605 will provide an introduction to complex analysis in several variables, including the topics mentioned above. Some details of the course may be adjusted according to the interests and backgrounds of the students enrolled.

No textbook required

****MATH 615 | Commutative Algebra 2 | Hochster, M. | MWF 2:00 PM - 3:00 PM**

Math 614

This is a topics course in commutative algebra which assumes as background an introductory course. Topics will include the structure theory of complete local rings, an introduction to homological methods, including a treatment of the functors Tor and Ext, Koszul homology, Grothendieck groups, regular sequences and depth, Cohen-Macaulay rings and modules, and the theory of flat homomorphisms, with applications to the method of proof by reduction to positive characteristic. Many open questions will be discussed.

There is no textbook: lecture notes will be provided.

****MATH 623 | Computational Finance | Feng, Q. | TR 8:30 AM - 10:00 AM**

This is a course on computational methods in finance and financial modeling. Using financial mathematics (like many branches of applied mathematics) in practice involves three tasks. First, one needs to develop mathematical models that accurately describe the real-life phenomena that one wishes to study – in the present case, probabilistic models for the evolution of prices, interest rates, and other relevant quantities. Once a model is chosen, the second task is to derive theoretical equations, or formulas, which establish relations between various objects in the financial markets: for example, the prices of derivative securities (options, bonds, etc), and the risk profiles of investment portfolios, as functions of risk factors. Finally, one needs to design and implement numerical methods to perform computations based on these formulas and equations.

This course is concerned with the latter task, and it has three components. In the first part, we will study the lattice (or, tree) methods, which correspond to the models based on discrete time Markov chains (e.g. the binomial model). We will discuss the pricing and hedging of financial derivatives in such models, using the arbitrage theory, or, more specifically, the risk-neutral pricing. We will, then, proceed to analyze the diffusion-based models of financial mathematics (including, e.g., the Black-Scholes model) and the associated Partial Differential Equations (PDEs).

We will discuss the finite difference methods, which provide numerical approximations for solutions to these PDEs. Both explicit and implicit schemes will be studied, the concepts of stability and convergence will be introduced, and a connection between the finite difference schemes and lattice methods will be established. After that, we will turn to the Monte Carlo simulations – the most general computational method for probabilistic equations. This method is based on generating a large number of paths of the underlying stochastic processes, in order to approximate the expectations of certain functions of these paths (which, e.g., may determine prices, portfolio weights, default probabilities, etc.). In addition to the standard Monte Carlo algorithms, we will study the variance reduction techniques, which are often necessary to obtain accurate results. The computational methods presented in this course will be illustrated using the popular models of equity markets (e.g. Black-Scholes, Heston), fixed income (e.g. Vasicek, CIR, Hull-White, Heath-Jarrow-Morton) and credit risk (e.g. Merton, Black-Cox, reduced-form models). In particular, we will cover certain deep learning algorithms for option pricing and introduce FBSDEs.

Required Textbooks: **Mathematics of Financial Derivatives: A Student Introduction** by Wilmott, P. ISBN: 9780521497893 & **Monte Carlo Methods in Financial Engineering** by Glasserman, Paul ISBN: 9780387004518

****MATH 626/ STATS | Probability and Random Processes II | Cohen, A. | TR 11:30 AM – 1:00 PM**

Advisory prerequisite: Math 625

A Ph.D. level course on stochastic processes in continuous time. Having at hand the intuition and the tools obtained in advanced probability and measure theory courses, we will study the theory of continuous-time stochastic processes and their applications. In the first half of the course, we will study the foundations of stochastic processes all the way from filtration to control theory. In the second half of the course, each student will present a paper from a given list. Specifically, we will cover the following topics:
1. Continuous-time processes (filtration, stopping-times, martingales, local-martingales, semimartingales, optional-stopping theorem, Doob's inequalities).
2. Brownian motion.
3. Stochastic integration (quadratic variation, stochastic integrals, Ito's rule, time-change, exponential martingales, Girsanov change of measure).
4. Stochastic differential equations (SDEs) (existence and uniqueness of strong solutions, weak solutions).
5. Stochastic control (dynamic programming principle, Hamilton-Jacobi-Bellman equation).
6. If time allows: mean-field games.

****MATH 628 | Machine Learning for Finance I | TBD | F 11:30 PM - 1:30 PM**

****MATH 632 | Algebraic Geometry II | Pixton, A. | TR 11:30 AM - 1:00 PM**
MATH 631 and Graduate standing. (3). (BS). May not be repeated for credit.

This is a continuation of Math 631. Topics will include sheaf cohomology, algebraic curves, differentials, and the Riemann-Roch theorem.

No textbook required

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****MATH 635 Differential Geometry Bieri, L. MWF 1:00 PM - 2:00 PM**
591 or equivalent. Consent of instructor required. (3). (BS). May not be repeated for credit.

In this course we will discuss important local and global aspects of differential geometry as well as the relation with the underlying topology. We will start with manifolds, connections, Riemannian metrics, curvature and the basic tools such as variational methods, Jacobi fields, and comparison theorems. Then we will continue to study sphere theorems, rigidity theorems and related topics. If time permits we will consider more advanced topics.

Required Textbook: **Riemannian Geometry**, by Manfredo Perdigão do Carmo, 2nd Edition, ISBN: 0817634908

****MATH 636 Topics in Differential Geometry Spatzier, R. TR 10:00 AM - 11:30 AM**
Topic: An Introduction to Dynamical Systems
Basic point set topology/manifold theory, real analysis

Goals: Dynamical systems is the study of the long-time behavior of diffeomorphims and flows. We will discuss ergodicity and mixing, and natural invariants such as entropy. This theory is particularly successful and interesting in the presence of a lot of hyperbolicity, i.e., when the flow or diffeomorphism expands and contracts tangent vectors exponentially fast. For these systems, we will then develop some of the crucial tools in Pesin theory such as unstable manifolds and Lyapunov exponents. We will illustrate the general theory by particular examples from geometry and dynamical systems on homogeneous spaces. I will apply these ideas to study some more general group actions, e.g., actions of higher rank abelian and semi simple groups. As was discovered in the last decade, these actions show remark- able rigidity properties, and sometimes can even be classified. These results have had important applications in other areas, e.g., geometry and number theory. While the material in the first part of the course is fundamental for many investigations in dynamics, geometry, several complex variables. the second half of the semester will bring us right to the forefront of current research.

Books: No text is required but the following are recommended**:**
Introduction to the Modern Theory of Dynamical Systems, Hasselblatt and Katok, Revised Edition, ISBN: 978-0521575577
Ergodic Theory, Einsiedler, Ward, Graduate Texts in Mathematics, ISBN: 978-0- 85729-021-2
An Introduction to Ergodic Theory, Walters, ISBN 978-0-387-95152-2

Talks: You are expected to give a talk during the semester.

Contact Info: 5850 East Hall, spatzier@umich.edu, 734 763 2192

****Math 651 Numerical Method I Viega, M.H. MW 10:00 AM – 11:30 AM**

Title: Topics in Deep Learning: Theory and Practice
Description: By now, deep learning has become a tool that permeates almost every scientific field in some capacity, although recent advances in theory for deep learning is not often studied in the classroom.

In this class, we will study deep learning through the lenses of mathematics and acquire practical know-how through hands-on tutorials. From the theoretical side, we will study recent approximation theorems and convergence results pertaining to deep neural networks, using tools from numerical analysis and probability theory. Other topics include: sampling strategies, optimisation, interpretability, convolutional neural networks, generative adversarial networks, encoder-decoders, reinforcement learning, physics informed neural networks.

From the practical side, you will learn a well-known framework (pytorch) with focus on topics of reproducibility, stability, reliability and convergence, emphasizing good development practices. By the end of the course, you will have had the chance to develop an end-to-end deep learning code.

A strong mathematical background in calculus and probability is expected. Experience in python is required.
Previous experience in machine learning is not required but might be helpful.

Grading: Homework and presentation.

Textbook:
Neural Networks and Numerical Analysis, Bruno Després, De Gruyter Series in Applied and Numerical Mathematics
Selected papers

****MATH 657 Nonlinear Partial Differential Equations Hani, Z. MW 2:30 PM - 4:00 PM**
MATH 656. (3). (BS). May not be repeated for credit.

Partial differential equations are at the core of models in science, engineering, economics, and related fields. These equations and their solutions have interesting structures that are studied by a beautiful combination of methods from analysis, geometry, probability, and other mathematical fields.

This course is an introduction to nonlinear partial differential equations and is a continuation of Math 656 from Fall 2022 (which is required as a prerequisite). It will start by introducing some analytical techniques that are central to the modern study of nonlinear PDEs like Sobolev spaces and some Littlewood-Paley theory. Afterwards, the course will present various techniques to construct and analyze solutions of nonlinear PDE. To showcase those tools and techniques, we shall use models coming from fluid equations (namely Euler and Navier-Stokes equations). Other models coming from nonlinear dispersive equations (the nonlinear Schroedinger and wave equations) may also be discussed.

****MATH 669 Topics in Combinatorial Theory Barvinok, A. TR 1:00 PM - 2:30 PM**
Topic: Combinatorics, Geometry, and Complexity of Integer Points

Good knowledge of linear algebra (3). (BS). May not be repeated for credit.

Integer points (points with integer coordinates) play an important role in algebra, number theory, combinatorics, and optimization. In this course, we will discuss some of the classical and recent results regarding integer points and lattices, such as:
- Minkowski Theorems and their applications in number theory and analysis,
- sphere packings and error-correcting codes, including recent advances on bounding sphere packing densities via Fourier analysis,
- transference and flatness theorems and their applications to Diophantine approximation and integer programming,
- Lenstra-Lenstra-Lovasz basis reduction algorithm and its applications to coding, polynomial factorization, etc.
- lattice-based "post-quantum cryptography" (that is, how to save e-commerce, should quantum computers be built)
- integer points in polytopes and the Ehrhart polynomial.

No textbook required

****MATH 681 Mathematical Logic Harrison-Trainer, M. TR 11:30 AM - 1:00 PM**

Mathematical maturity appropriate for a 600-level MATH course. Graduate standing. (3). (BS). May not be repeated for credit.

Mathematical logic is the study of mathematics itself as a formal process where theorems, written down in a formal language, are proved from assumptions using formal deductive principles. The first part of this course will set up the basic ideas, defining the formal language of first-order logic in which we can write down mathematical statements and writing down a set of deductive rules which we can use to build valid proofs. These mathematical statements are supposed to describe mathematical structures, and so we will also define these and say what it means for a statement to be true of a particular model. We will show that if we prove something, then it must be true (soundness). This means that our rules of deductive reasoning are all correct. On the other hand, we might wonder whether we might be missing any deductive rules; we will show that everything true is provable (completeness), which means that our deductive rules are sufficient for all reasoning.

In the rest of the course, we will learn many of the central tools of logic which are not only useful in all areas of logic, but also give a useful perspective on other areas of mathematics such as algebraic geometry. The main phenomenon here is that if we have any non-trivial model, there will be other non-isomorphic models which satisfy all of the same first-order sentences as our given model. By studying these non-standard models, we can learn something about our original model. The main theorem here is the compactness theorem, which says that if we have an infinite set of sentences, and any finite set is consistent (has a model), then the whole set is consistent.

Other topics we will cover may include: quantifier elimination, which is a generalization of Chevalley's theorem on constructible sets; decision problems and interpretations; back-and-forth arguments, or why the rationals are the only countable dense linear order without endpoints; ultraproducts, which let us build copies of the reals with infinitesimal elements; types and the topology of the type space; interpolation theorems; indiscernibles and Ramsey's theorem; and amalgamations of structures and Fraisse Limits.

The only official prerequisite will be mathematical maturity appropriate for a 600-level course, but we will frequently use examples coming from algebra and analysis (e.g., rings and fields). Students unsure whether they are adequately prepared for the course are encouraged to write to the instructor. No textbook is required.

Optional Texts: **Fundamentals of Mathematical Logic**, by Hinman & **Model Theory**, by Marker

****MATH 697 Topics in Topology Canary, R. MWF 2:00 PM - 3:00 PM**

Topic: Deformation Spaces of Geometric Structures

Graduate standing. (2 - 3). (BS). May not be repeated for credit.

One of the most classical deformation spaces of geometric structures is the Teichmuller space of hyperbolic structures on a fixed closed surface. We will begin by reviewing the theory of Teichmuller space and then move on to discuss other deformation spaces whose study is inspired by the study of Teichmuller space. These spaces include the space of convex cocompact hyperbolic structures on a n-manifold with boundary, the space of strictly convex real projective structures on a closed manifold, and the space of Hitchin representations of a closed surface group into PSL(n,R). All of these theories, including Teichmuller theory are special cases of the theory of Anosov representations of a hyperbolic group into a semi-simple Lie group.

The only prerequisites for the course are the material on covering spaces in Math 592. Lecture notes for much of the material are available on my webpage. The pace of the course, and which precise topics are covered, will be determined in discussion with the participating students.

****MATH 710 Topics in Modern Analysis II Chelkak, D. TR 1:00 PM - 2:30 PM**

Topic: Conformal Invariance in 2D Statistical Physics

Complex Analysis and Probability (incl. martingales), e.g. MATH 596 & 625. *(3). (BS). May not be repeated for credit.*

This course is an introduction to conformal invariance emerging in two-dimensional statistical physics models at the critical temperature. It is based on an interplay of complex analysis and probability, with a glimpse at conformal field theories coming from theoretical physics as the exposition progresses. The course includes an introduction of basic objects in this research area in full rigour and a less formal discussion of more involved concepts/results coming from recent developments.

Topics include (some choices can vary depending on the interests of the enrolled students): - Brownian motion as the limit of random walks and its conformal invariance; - Uniform spanning trees (UST) and loop-erased random walks (LERW); - More examples: self-avoiding walks, Ising model, FK-percolation; - Schramm-Loewner Evolutions (SLE): definition and basic properties; - Percolation: convergence to SLE(6); - Gaussian Free Field (GFF) and Brownian loop soups; - Bipartite dimer model: convergence to the GFF; - Ising model: free fermions and convergence results; - [time permitting] Conformal Field Theory description of the critical Ising model; - [time permitting] Conformal loop ensembles, level lines of the GFF and Imaginary Geometry.

No textbook required

****MATH 732** **Topics in Algebraic Geometry II** **Mustata, M.** **TR 1:00 PM - 2:30 PM**
***Topic:* D-modules and Singularities**

MATH 631-632 or equivalent. (3). (BS). May not be repeated for credit.

The goal of the course is to introduce D-modules and mixed Hodge modules and discuss applications to singularities. The theory of D-modules (a.k.a. algebraic analysis) studies modules over ring of differential operators. While it can be viewed as an algebraic approach to linear PDEs, it found many connections to algebraic geometry, singularity theory, and representation theory. On the other hand, Saito's theory of mixed Hodge modules provides a functorial framework for doing Hodge theory, in which the coefficients are certain filtered D-modules. The first part of the course will introduce D-module theory, the second half will give an overview of mixed Hodge modules, and the third part will be devoted to invariants of singularities that can be defined and studied in the framework of these theories.

No Textbook Required

****MATH 777** **Diphantine Problems** **Zou, J.** **TR 2:30 PM – 4:00 PM**

This course gives an introduction to the theta correspondence and its applications to number theory and representation theory.

The theta correspondence was used by H. Maas and others in special cases but was formally organized into a theory by R. Howe in the 1970s. This was built upon the work of A. Weil in the 1960s which provided a representation theoretic treatment of theta functions via his construction of the so-called Weil representations.

We will discuss both the local theta correspondence and automorphic theta lifting. Topics to be covered include:
--Stone–von Neumann theorem
--Different models of the Weil representation
--Howe duality theorem
--Siegel-Weil formula
--Periods and the theta correspondence
--Applications: Theta series and Siegel modular forms

Prerequisites: Some basic knowledge on representation theory and modular forms.
No textbook is required. References will be provided during the course.