Cardinal Characteristics Database

Zach Norwood & Iian Smythe

Project Description: Cantor's work showed that there are uncountable sets, infinite sets that are strictly larger than the set of natural numbers. But are there infinite cardinalities that are almost countable, in the sense that some theorems about the countable might extend to these cardinalities? Sometimes yes, as it turns out, and cardinal characteristics of the continuum provide a framework for studying such things.

Our understanding of cardinal characteristics has improved in exciting ways even in the last ten years. Some of the latest developments include the proof (Malliaris–Shelah, PNAS 2013) that $p = t$ and the recent construction (Goldstern–Kellner–Shelah, Annals of Math. 2019) of a model of 'Cichon's Maximum'. (It is not important that you understand what these things mean, but you will after the project!)

This project will categorize the cardinal characteristics and known relations between them in a user-friendly database or wiki. Visualizing relations among the characteristics would be useful to mathematicians, so one important goal of the project involves turning a directed graph into an appealing picture.

After completing this project, students should appreciate a beautiful area of modern set theory and have mastered some database and graph- visualization tools.

Prerequisites: Math 217, some coding experience, and an analysis course (math 297, 351, or 451). Note Math 451 is preferred among the analysis courses. Math 582 would be great, but is certainly not required.

Conway's Game of Life

Katie Storey & Jörn Zimmerling

Project Description: In this LoG(M) project, we will study Conway's Game of Life, a 2D cellular automaton. The rules of Conway's game of life are deceptively simple. In a two dimensional network of cells, a cell can be alive or dead:

1. A living cell with two or three living neighbors survives to the next time step
2. A cell with no or one living neighbor dies
3. A dead cell with exactly three neighbors will be alive in the next time step

These rules seem to balance the population and allow for interesting dynamics. We will implement our own Game of Life and study patterns and behavior of this system. After studying the many life forms that can support themselves in this rule set, we will investigate a few questions:

- What are "good" rules for a 3D version of Game of Life?
- Can we build simple logic gates in Game of Life?
- Can we find new stable configurations or oscillators?
- Can we find alternative rules in 2D with similar properties as Conway's rules?
- Can we come up with a probabilistic version of Game of Life?

Prerequisites: Knowing a programming language will help us get the project of the ground quickly. Having taken a flavor of linear algebra will help but is not required.

Gerrymandering & Visualization

Samuel Hansen & Tim Ryan

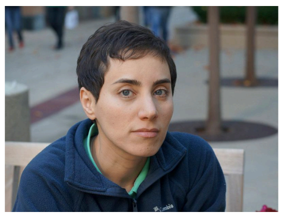
Project Description: There has been a recent boom in interest in using mathematics to investigate gerrymandering and multiple research groups have been formed such as the Metric Geometry and Gerrymandering Group Redistricting Lab out of Tufts and Quantifying Gerrymandering at Duke. These groups have helped create a number of new ways to detect and measure gerrymandering and its potential effects on election. The students in this project will be provided with a survey of these results and be guided toward a deeper understanding of the core ideas and techniques.

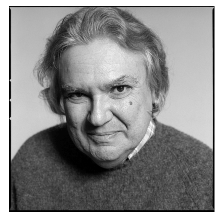
Students will then choose a technique or two and apply them to election data to help develop and understanding of the state of redistricting in Michigan and potentially other states. Part of this project will be to create an interactive website which will include visualizations of the results of the mathematical gerrymandering measurement techniques.

Prerequisites: Coding experience

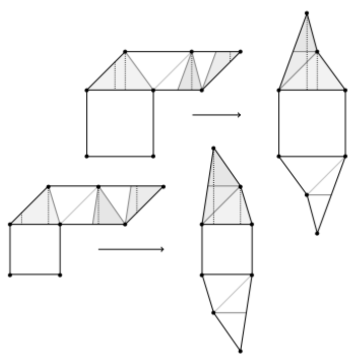
Convex Presentations of Translation Surfaces

A translation surface is a collection of polygons in the plane with sides identified by translation. These seemingly simple objects have been at the forefront of modern mathematics, being the main object of study in the work for which Maryam Mirzakhani won the 2014 Fields Medal and Alex Eskin the 2020 Breakthrough Prize.





A translation surface is said to have a convex presentation if, up to cut and paste, the translation surface can be presented as a single convex polygon in the plane with opposite sides identified.



Questions

(1) Are there genus three translation surfaces with more than one convex presentation? More than 8?

(2) What does the "moduli space" of all convex polygons look like? What about the moduli space of convex polygons with area one? What is the volume of this moduli space?

(3) A translation surface is called square-tiled if the polygons that constitute it can be taken to be square. Which genus three square-tiled surfaces have a convex presentation?

(4) Using the Kenyon-Smillie invariant determine the Veech surfaces that have a convex presentation in genus three.

Flowing through confined geometries

Hanliang Guo

Background Suspensions of rigid and/or deformable particles in viscous fluids flowing through confined geometries are ubiquitous in natural and engineering systems and the shapes of these confined geometries are notoriously complex. For example, the spermatozoa need to navigate the female reproductive tract (and do it efficiently) to reach the ovum cell; carefully designed micro-fluidic chips can be used to sort and separate cells based on their mechanical properties (size, shape, rigidity etc.).

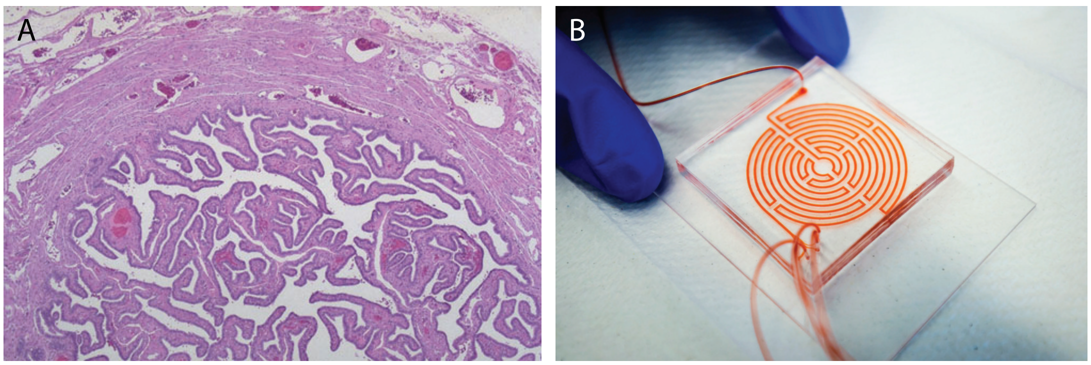


Figure 1: A: The microscopic image of a cross section of the Fallopian tube. B: Blood samples run through a microfluidic chip. (cr: https://news.engin.umich.edu/photography/labyrinth-chip/)

Goals In this project, we will numerically study the shape effects of the confined geometry on the motions of the suspensions. In particular, we will start with existing computer programs to investigate the flows of suspensions in pressure-driven pipes with different geometries (sample result shown in Fig. 2); we will then modify the programs to implement different boundary conditions to emulate different biological scenarios (e.g., slip boundary conditions for cilia-driven flows). You will be writing your own codes (simple and challenging), working with cutting-edge computational programs, and get a flavor of the University's supercomputer (Great Lakes).

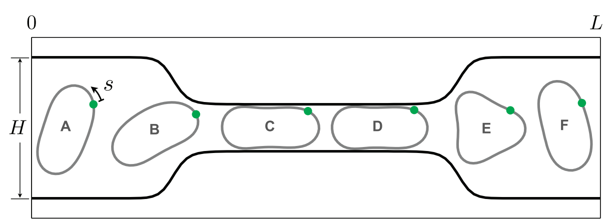


Figure 2: Dynamics of a 2D vesicle through a channel with a constriction. [O. Pak *et al.* PNAS (2015)]

Prerequisite Linear algebra (Math 214 or equivalent) and Differential equations (Math 216 or equivalent). Experience with scientific computing (Math 371 or equivalent) and/or Matlab will be nice.