Orthogonal Polynomials

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Project Description

Asymptotic analysis of orthogonal polynomials crops up in plenty of fields, and there is no shortage of their applications in approximation theory, mathematical physics, and random matrix theory among other fields. However, unlike classical orthogonal polynomials, polynomials orthogonal with respect to complex-valued weights may "degenerate". More precisely, for a complex weight $\rho(z)$, a polynomial $P_n(z)$ satisfying n orthogonality conditions

$$\int_\gamma z^k P_n(z)\rho(z)\mathrm{d}z = 0 \quad \text{for} \quad k = 0, 1, \cdots, n-1,$$

may have degree less than n. This degeneration can be a serious obstacle that often presents itself in subtle ways in the analysis.

Guided by the theory available, the objective of this project is to explore some specific families of orthogonal polynomials and compute examples of families exhibiting interesting degeneration patterns. Roughly speaking, the plan is to

1. learn what non-Hermitian orthogonal polynomials (OPs) are and how to compute them and their zeros,
2. begin computing OPs in symmetric situations where degeneration may occur while exploring how to do so most effectively,
3. compute OPs dependent on auxiliary parameters and consider how a polynomial may experience degeneration with choice of parameter(s),
4. explore ways to visualize this information in a reasonable manner.

Prerequisites

Math 217 and Math 351/451 or equivalents. A course in complex analysis would be nice, but not required.