Cardinal Characteristics Database

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Project Description: Cantor's work showed that there are uncountable sets, infinite sets that are strictly larger than the set of natural numbers. But are there infinite cardinalities that are almost countable, in the sense that some theorems about the countable might extend to these cardinalities? Sometimes yes, as it turns out, and cardinal characteristics of the continuum provide a framework for studying such things.

Our understanding of cardinal characteristics has improved in exciting ways even in the last ten years. Some of the latest developments include the proof (Malliaris–Shelah, PNAS 2013) that $p = t$ and the recent construction (Goldstern–Kellner–Shelah, Annals of Math. 2019) of a model of 'Cichon's Maximum'. (It is not important that you understand what these things mean, but you will after the project!)

This project will categorize the cardinal characteristics and known relations between them in a user-friendly database or wiki. Visualizing relations among the characteristics would be useful to mathematicians, so one important goal of the project involves turning a directed graph into an appealing picture.

After completing this project, students should appreciate a beautiful area of modern set theory and have mastered some database and graph- visualization tools.

Prerequisites: Math 217, some coding experience, and an analysis course (math 297, 351, or 451). Note Math 451 is preferred among the analysis courses. Math 582 would be great, but is certainly not required.