Van Eenam Lecture Series

November 14, 15 & 16, 2023

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at Princeton University

Lecture I: Mean Field Games and Kuramoto Synchronization

TUESDAY, NOV, 14, 2023 | 4:00 PM | 1360 EAST HALL

The main goal of this talk is to introduce an exciting new area of mean field games modeling interactions between large number of identical particles. In this formalism, instead of positing the dynamics of the individual particles, one lets them endogenously determine their behaviors by minimizing a given cost functional and hopefully, settling in a Nash equilibrium. Initiated by Larry & Lions, and Huang, Malhame, & Caines in 2006, mean field games has found an amazing range of applications. This talk uses the specific example of classical Kuromato synchronization to introduce the novel approach and its potential. Originally motivated by systems of chemical and biological oscillators, the Kuramoto system is the key mathematical model to describe self organization in complex systems. These autonomous oscillators are coupled through a nonlinear interaction term which plays a central role in the long term behavior of the system. While the system is unsynchronized when this term is not sufficiently strong, fascinatingly, they exhibit an abrupt transition to a full synchronization above a critical value of the interaction parameter. Mean field approach also delivers same type of results including the phase transition from incoherence to synchronization.

Lecture II: Computing Free Boundaries by Neural Networks and Simulations

WEDNESDAY, NOV. 15, 2023 | 4:00 PM | 1360 EAST HALL

This talk discusses a numerical method for the computation of free boundaries when a stochastic representation is available. It is based on an algorithm which we call deep empirical risk minimization developed by E, Han & Jentzen. Their approach applies generally to many stochastic optimal control problems. In the presence of free boundaries, it has to be modified to account for training based on hitting times. In this talk, I outline how this is achieved for the classical problems of optimal stopping or the obstacle problem, and for the Stefan problem for the water-ice interfaces. For the Stefan problem, we use the recent stochastic representations, the notion of physical probabilistic solutions, and level-sets parameterized by deep neural networks on the numerical side.

Lecture III: Eikonal Equations on Wasserstein Spaces

THURSDAY, NOV. 16, 2023 | 3:30 PM | 4448 EAST HALL

Mean-field or McKean-Vlasov type optimal control is closely related to the exciting program of mean-field games. Dynamic programming approach to these control problems result in nonlinear partial differential equations on the space of probability measures. These equations not only require the solution to be differentiable but impose further regularity on the derivatives which are being on the dual of the set of measures are also functions themselves. Despite these difficulties, several approaches to characterize the value function of the control problems as the unique appropriate weak solutions have been developed. In this talk, I discuss a comparison result between sup and super viscosity solutions of the associated dynamic programing equations. Main technical result uses negative Sobolev norms and the classical techniques from the viscosity theory.

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