RESEARCH PROJECTS

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This research topic involves developing the method of *finite difference (discrete) approximations* for optimization problems, implementing numerical algorithms, and applying them, especially, to the calculations in machine learning, deep reinforcement learning, artificial intelligence, and their applications. Below, I briefly discuss some of the main ideas and sketch some applications for the future research.

1 Model Predictive Control

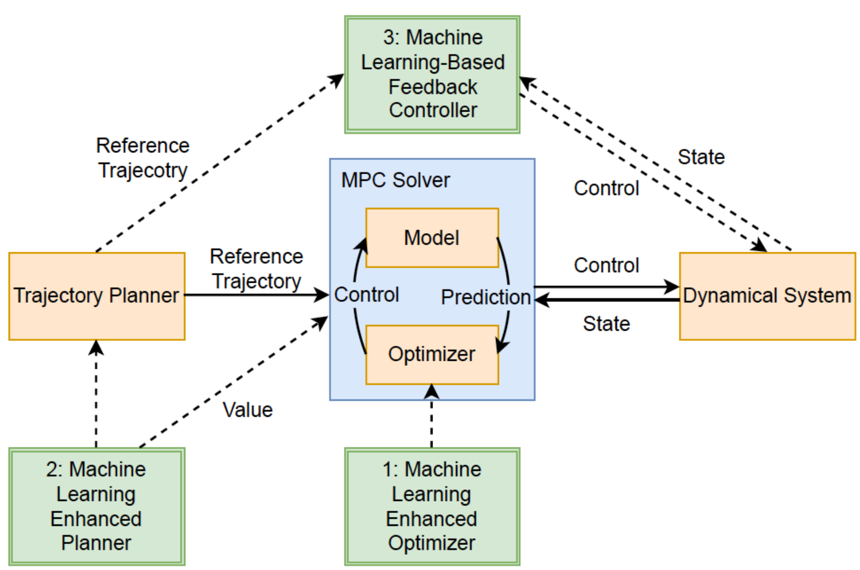
Solving complex optimal control problems have confronted computational challenges for a long time. Recent advances in machine learning have provided us with new opportunities to address these challenges. This project takes model predictive control, a popular optimal control method, as the primary example to survey recent progress that leverages machine learning techniques to empower optimal control solvers. We also discuss some of the main challenges encountered when applying machine learning to develop more robust optimal control algorithms.

Our approach is based on developing the method of discrete approximations. We derive the necessary optimality conditions of the discrete Euler-Lagrange type and the *numerical algorithms* to compute the optimal solution to the optimization and optimal control problems in robotics. Our further research goals concerning this model include developing efficient numerical algorithms (and coding in Python and Matlab) to solve the optimal control problems for them with large numbers of robotics in the corresponding models. It could be done, in particular, by using an appropriate discretization and employing numerical algorithms of finite-dimensional optimization to the discrete-time problems obtained in this way.

In this project we formulate the *optimal control problem* of type (P) that can be treated as a continuous-time counterpart of the discrete algorithm. Consider the cost functional

$$\text{minimize} \quad J[x,u] := \frac{1}{2}\big\|x(T)\big\|^2, \tag{1.1}$$

We describe the continuous-time dynamics by the controlled process and the dynamic noncollision condition $\|x^i(t) - x^j(t)\| \geq 2R$. Next, applying the necessary optimality conditions for the controlled model allows us to obtain the optimal solution for this model.



We derive a new *discrete approximation method* to obtain the necessary optimality conditions for optimal control problem and new *numerical algorithms* to find the optimal solution using the obtained conditions.

2 Applications

- Students can implement the code for the robot model in the case n is a large number using the necessary optimality conditions that we obtained. We propose to use the *discrete approximation method* to approximate solution of the optimal control problems. Students can try to construct a numerical scheme for some well-known robotic models. It is also worth comparing the rate of convergence between the new method and the traditional one.

- We also propose a machine learning enhanced algorithm for solving the optimal landing problem. Using Pontryagin's minimum principle, we derive a two-point boundary value problem for the landing problem. The proposed algorithm uses deep learning to predict the optimal landing time and a space-marching technique to provide good initial guesses for the boundary value problem solver. The performance of the proposed method is studied using the quadrotor example, a reasonably high dimensional and strongly nonlinear system. Drastic improvement in reliability and efficiency is observed.

Such those projects can benefit the students in multiple ways. First, they are introduced new areas related to optimization and optimal control. Second, some well-known models help them to engage mathematics in real-world issues. Finally, it would also give the student an opportunity to gain a new skill by learning a programming language such as Matlab and Python to implement the numerical methods needed to verify the approximation's accuracy.

3 Prerequisites

- Coding skills in Matlab or Python.
- Courses or Tests:
  - Calculus classes: I, II, III, IV/ Minimum Grade of B/ May not be taken concurrently.
  - Linear Algebra/ Minimum Grade of B/ May not be taken concurrently.
  - Real Analysis/ Functional Analysis (Optional).