Syllabus for the AIM Preliminary Examination in Probability & Discrete Mathematics

Algorithms

(Much of Chapters 2,3,4,5,6,7,8, and 13 of Kleinberg and Tardos.)

1. Basics
   - Computational tractability
   - Asymtotic order of growth
   - Common running times: $\log(n), \log^{O(1)}(n), O(n), n^{O(1)}, 2^{O(n)}, n!, 2^{n^{O(1)}}$.
   - Basic data structures: lists, arrays, priority queues
2. Graphs (see also Combinatorics syllabus)
   - Defintions
   - Directed and undirected graphs; directed acyclic graphs and topological ordering
   - Connectivity and traversals
3. Greedy algorithms
   - Overall approach
   - Examples: scheduling, shortest paths, minimum spanning trees, Huffman codes
4. Divide and conquer
   - Master theorem (from Cormen, Leiserson, Rivest, and Stein)
   - Merge sort
   - Fast Fourier transform
5. Dynamic programming
   - Overall approach; memoization
   - Examples: sequence alignment (longest common subsequence), shortest paths in a graph
6. Network flow
   - Ford-Fulkerson algorithm
   - Bipartite matchings
7. NP
   - NP definitions; hardness, completeness, reductions
   - Reductions among problems; satisfiability, coloring, partitioning.
8. Randomized algorithms (See also Probability section.)
   - Chernoff bounds
   - Examples: median, quicksort, hashing, MAX3SAT

Probability

(Much of chapters 1–6 of Ross. See also Combinatorics section.)

1. Axioms of probability
    - Sample spaces, events
2. Conditional probability
    - Bayes's formula
    - Independent events
3. (Discrete) Random variables
    - Expectation and variance; relationship with sums of random variables
    - Common random variables: Bernoulli, binomial, Poisson, geometric
    - Cumulative distribution functions
4. Continuous random variables
    - Expectation and variance
    - Examples: uniform, normal, exponential
    - Normal approximation to binomial; central limit theorem
    - Distribution functions
5. Jointly distributed random variables
    - Joint distribution functions
    - Independence
    - Sums of independent random variables
    - Conditional distributions

Combinatorics

(Much of chapters 1–7 and 11 of Brualdi; basic error-correcting codes)

1. Permutations and combinations
    - Basic counting principles
    - Functions, sets and multisets
2. Pigeonhole principle and Ramsey's theorem
3. Generating permutations and combinations
    - Inversions
    - Partial orders and Hasse diagrams
4. Binomial coefficients
    - Pascal's triangle
    - Binomial theorem
    - Unimodality of binomial coefficients

- Multimonial theorem

5. Inclusion-exclusion principle
    - Combinations with repetition
    - Derangements
    - Permutations with forbidden positions
    - Möbius inversion
6. Recurrence relations and generating functions
    - Exponential generating functions
    - Solving homogeneous and simple inhomogeneous recurrence relations
7. Graph theory
    - Euler paths and cycles
    - Hamiltonian paths and cycles
    - Trees
    - Coloring
    - Planarity
    - Independence number
8. Error-correcting codes
    - Linear error-correcting codes; generators and parity-check matrices
    - Hamming, Hadamard, Reed-solomon, and Reed-Muller codes

Miscellaneous

1. Finite fields of prime order
2. Basic linear algebra (solving systems, rank, etc.), including over finite fields.