AIM Preliminary Exam: Advanced Calculus & Complex Variables

January 6, 2014

There are five (5) problems in this examination.

There should be sufficient room in this booklet for all your work. But if you use other sheets of paper, be sure to mark them clearly and staple them to the booklet.

Problem 1

Let $f_1, f_2, \ldots$ be integrable real-valued functions on $\mathbb{R}$. Suppose that

$$\int_{-\infty}^{\infty} |f_n(x)|\, dx \to 0 \quad \text{as } n \to \infty.$$

Prove or provide a counterexample to each of the following statements.

(a) If each function f_n is continuous, then $\lim_{n\to\infty} f_n(x) = 0$ for every $x \in \mathbb{R}$.

(b) If each function f_n is uniformly continuous, then $\lim_{n\to\infty} f_n(x) = 0$ for every $x \in \mathbb{R}$.

Problem 1

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Problem 2

Let $\{a_n\}$ be the sequence of real numbers defined by the conditions:

$$a_0 = 1, \quad a_1 = 4, \quad \text{and} \quad a_n = a_{n-1} + a_{n-2} \quad \text{for } n \geq 2\,.$$

Determine the radius of convergence of the power series

$$\sum_{n=0}^{\infty} a_n z^n.$$

Problem 2

Problem 2

Problem 2

Problem 3

Compute

$$\int_0^{2\pi} \exp(e^{2it} - 3it)\, dt.$$

Problem 3

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Problem 4

Find the maximum and minimum values of the function $f(x, y) = 3x + y$ on the circle $x^2 + y^2 = 10$.

Problem 4

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Problem 5

Let T be the trapezoid in $\mathbb{R}^2$ with vertices $(1,0)$, $(2,0)$, $(0,-2)$ and $(0,-1)$. Evaluate the integral

$$\int_T e^{(x+y)/(x-y)} ds(x,y)$$

over the region T using the change of variables $(x,y) \to (u,v)$, where

$$u = x + y, \qquad v = x - y.$$

Sketch T in each of the x-y plane and the u-v plane.

Problem 5

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