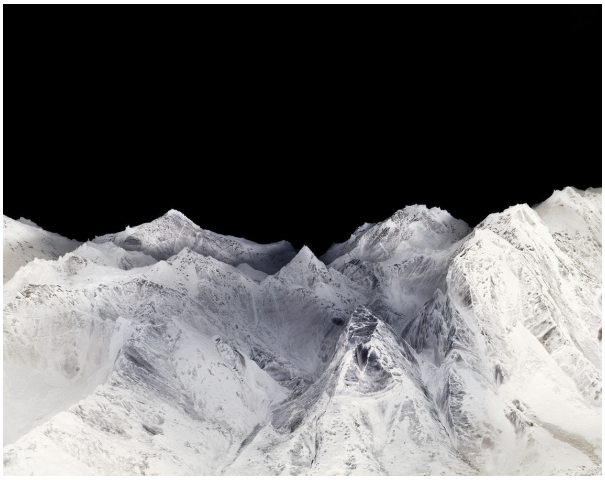
Fractional Brownian Motions

Hai Le

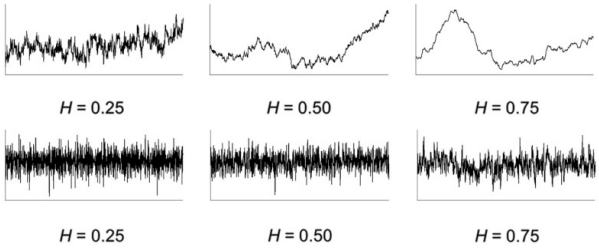
1 The project

The term *Fractional Brownian motions* (fBm's) were first used by Mandelbrot and van Ness in their seminal paper "*Fractional Brownian motions, fractional noises and applications*" to give a generalization to the standard Brownian motions. They realized fBm's sometimes serve as a better model for many random phenomena because of its "interdependence between distant samples". What that means informally is the change in the future will depend on the changes in the past, unlike in the case of standard Brownian motions where the changes are independent of each other. Consequently, fBm's have found applications in modeling random processes appearing in economics, finance, hydrology, wave propagation in random media, etc.

The goal of this project is to introduce its participants to the field of ***stochastic processes***. In particular, we will focus on fractional Brownian motions (and standard fBm's). On the theoretical side, we will learn how to construct fBm's and study their properties under the framework of Gaussian processes. On the numerical side, we will learn how to simulate fBm's and as a prize, use that to draw moutains!



(a) Mountains generated by fBm.[1]



(b) Realizations of fBm's at different Hurst exponents.

2 Prerequisites

- Math 217 Linear Algebra or equivalent
- Some familiarity with probability theory (ideally 425: Introduction to probability or equivalent).
- Some coding experience. We will mainly use Python and Matlab.

[1] The Book of Shaders by Patricio Gonzalez Vivo & Jen Lowe.