LOG(M) PROJECT: BESSEL FUNCTIONS FOR $\mathrm{GL}_n(\mathbb{F}_q)$

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Many special functions in analysis have analogs over finite fields, which occur naturally in representation theory. Some of these functions include the beta function, the gamma function, and the Bessel function.

Gelfand and Graev introduced a generalization of the Bessel function, associated to every irreducible representation of $\mathrm{GL}_n(\mathbb{F}_q)$. This Bessel function has great importance in representation theory of $\mathrm{GL}_n(\mathbb{F}_q)$. However, it is quite difficult to compute its values by hand.

The goal of this project is to teach the students how to use SageMath in order to solve problems in abstract math. In this project, the students will compute Bessel functions for irreducible generic representations of $\mathrm{GL}_n(\mathbb{F}_q)$. The computations will be symbolic computations which will be performed using SageMath. The students will learn some basic representation theory of finite groups and some basic knowledge about $\mathrm{GL}_n(\mathbb{F}_q)$ and its representation theory. One of the first tasks the students will perform is compute the Bessel function for $\mathrm{GL}_2(\mathbb{F}_q)$ and see that it is in fact analogous to the classical Bessel function.

Prerequisites.
- Some familiarity with Group Theory (ideally Math 412 Introduction to Modern Algebra).
- Math 217 Linear Algebra or equivalent (ideally Math 420 Advanced Linear Algebra).
- Some coding experience - ideally Python, because SageMath is based on Python, but any other past coding experience would also work.