

Chow functions of unbounded posets

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Overview Chow polynomials are Hilbert–Poincaré series of the Chow rings of toric varieties associated to matroids. Ferroni, Matherne and Vecchi showed that the Chow polynomial of a matroid is a special case of a broader construction: every bounded, locally finite poset with a weak rank function and a kernel carries a Chow function, assigning a palindromic polynomial to each interval. Many posets in arrangement theory are unbounded: intersection posets of affine arrangements are semilattices, posets of layers of toric and elliptic arrangements need not be semilattices at all, and some are infinite. The Chow function still makes sense on each interval, but there is no accepted notion of the Chow polynomial of such a poset, and their geometric properties and positivity properties are unexamined.

Questions Compute and study some Chow functions of unbounded posets from the literature: geometric semi-lattices, the locally geometric and supersolvable posets of Bibby–Delucchi, Dowling posets, and posets of layers. What positivity properties do we observe? For a geometric lattice, the Chow function recovers the Hilbert–Poincaré series of the Chow ring of the matroid, or equivalently, of the toric variety of its Bergman fan. Does anything similar hold for the arrangements whose intersection posets are unbounded above?