

A sphere by any other name

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In the most familiar case, a sphere is the set of points in $\mathbb{R}^3$ satisfying the equation $x^2 + y^2 + z^2 = 1$, which we can also describe as the set of vectors $\vec{v}$ so that $\langle \vec{v}, \vec{v} \rangle = 1$. We can ask many questions about the sphere; what are the motions of $\mathbb{R}^3$ that preserve it? What are all the different ways that a circle can live inside a sphere? What do those ways tell us about the geometry of the sphere?

We can also take another avenue: I would like us to explore in this project. What if we consider a “sphere” in a vector space equipped with a different bilinear form? These spheres can look very different and have different geometric properties. Sometimes, there is also a good notion of “positivity,” or of a “cyclic order,” on them or their boundaries. In this project, we will learn a bit about these different spheres, and figure out some ways to visualize this mysterious order.

Prerequisites. You should be familiar and comfortable with linear algebra. Knowing some multivariable calculus helps. Topology is useful for motivation, but is not required.