Evolving curves using differential equations

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Geometric flows are differential equations that continuously evolve shapes such as curves in the plane, surfaces in 3D space, or the metric of a space. In recent years they have attracted interest across both pure and applied mathematics. On the pure side, perhaps the most well-known example is the Ricci flow, introduced by Hamilton and used by Perelman to prove the Poincaré conjecture. On the applied side, these equations describe the motion of interfaces and grain boundaries in material science.

In this project we will focus on the simplest geometric flow, the *curve shortening flow*, in which a curve in the plane evolves in the direction of its curvature. One can think of curve shortening flow as a "heat equation for the curve itself" which smooths and regularizes curve geometry. Despite its simplicity, this flow exhibits much of the prototypical behaviour associated to geometric flows. Our goal will be to understand some of these phenomena from a theoretical standpoint while also implementing a numerical algorithm for visualizing curve shortening flow. Given more time, we could also explore other variants of geometric flows.

Prerequisites. Multivariable calculus and Ordinary differential equations (a first course, no PDE background assumed). Real analysis is highly recommended for the more theoretical aspects. Python programming (familiarity with Numpy and matplotlib would help but not necessary).

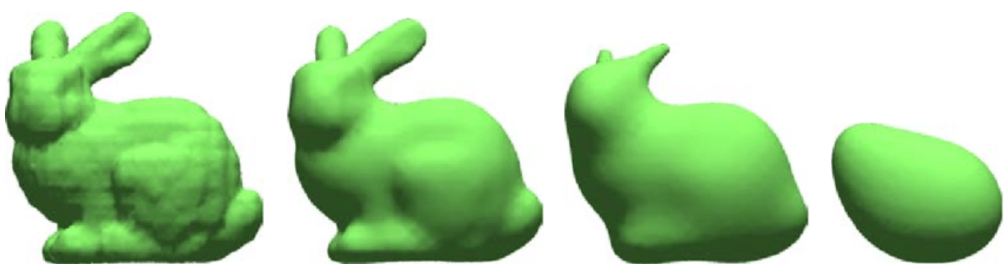

Figure 1: Mean curvature flow, a 2D analogue of what we will be studying. Taken from [3].

References

[1] Ben Andrews, Bennett Chow, Christine Guenther, and Mat Langford. *Extrinsic geometric flows*, volume 206. American mathematical society, 2022.

[2] Robert Haslhofer. Lectures on curve shortening flow. *arXiv preprint arXiv:2604.01981*, 2026.

[3] Simon Vey and Axel Voigt. Amdis: adaptive multidimensional simulations. *Computing and Visualization in Science*, 10(1):57–67, 2007.