Stochastic Control in Financial Decision Making

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1 Introduction

We aim to investigate the mathematical principles behind decision-making under uncertainty, focusing on applications in financial mathematics. The central theme is stochastic control: how to choose an optimal action in dynamic systems governed by randomness? By the end of this project, students will have a solid understanding of stochastic control theory, its applications to finance, and the main numerical algorithms to approximate optimal decisions.

2 Project objectives

1. **Stochastic control theory:** We will start with the theory of stochastic control, from learning about stochastic differential equations and the proper tools to treat them, to studying the dynamic programming principle, deriving the Hamilton–Jacobi–Bellman equation and its verification. We will apply these tools to real-life inspired applications such as the Merton portfolio optimization problem and the optimal consumption problem.

2. **Classical numerical methods:** The second part of the project will focus on numerical implementation. Students will use PDE solvers and stochastic simulations to solve the problems studied in theory—first in one dimension, and then in higher-dimensional settings. We will

also numerically investigate features such as portfolio constraints and transaction costs, which are difficult to handle analytically but have important practical implications.

3. **Applications to finance and reinforcement learning:** Should time allow, we will investigate the connection between reinforcement learning and stochastic control. Using the Merton's problem as an example, we can formulate it as an episodic RL problem. We can then implement some suitable RL algorithms, either in a discrete or continuous state/action space, and train the RL agents to learn investment policies through simulation. And finally, we can compare the learned policies with known solutions.

3 Prerequisites

Probability theory (525), basic knowledge of MATLAB or Python, and interest in finance. Mathematics of finance (423) and Stochastic Analysis with financial applications (474) are recommended but not required.