Realizing Transfer Systems

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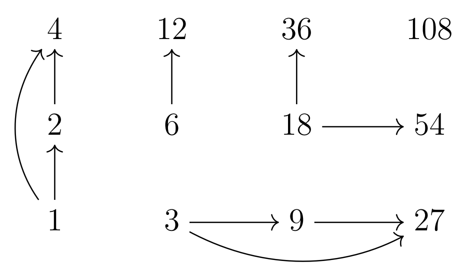
For a positive integer n, take any non-empty collection S of integers k satisfying $0 \le k < n$ you want. From S we obtain a relationship $\to$ called the corresponding *transfer system* between the factors of n. It is defined by having $d_1 \to d_2$ whenever

- d_2 is divisible by d_1 and
- if x is any number in S and $y - x$ is divisible by d_2, then there is a z in S so that $z - y$ is divisible by d_1.

When a relation $\to$ arises from S this way, S is said to *realize* the transfer system $\to$.

The purpose of this project is to explore questions regarding the possible transfers that arise as we vary S. If there is an S for a certain transfer relation (like the one pictured below for $n = 108$), which factors of n appear as greatest common factors of elements of S with n?



Transfer systems describe potential systems of algebraic operations in equivariant homotopical algebra that include symmetric multiplication operations alongside ordinary multiplication. The realizable transfer systems are those that can be described using linear algebra and have a clearer relationship to geometry.

No equivariant homotopical algebra is needed for this project, but familiarity with modular arithmetic and some coding experience will be helpful. For even deeper explorations, some familiarity with finite abelian groups and experience with dynamic programming would be helpful.