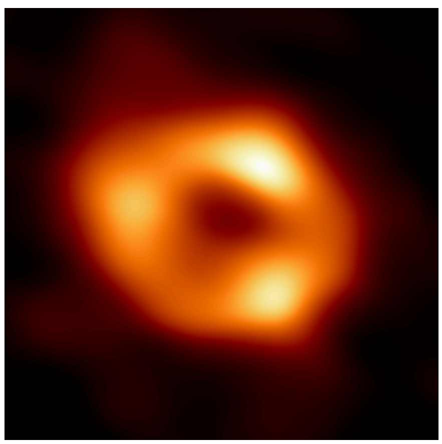
Gravitational Lensing by a Naked Singularity

James Wheeler

Knowledge of how gravity deflects light is fundamental to the interpretation and understanding of astrophysical observations, and the mechanics of lensing underlie some of the most dramatic evidence for Einstein's General Relativity, the modern theory of gravity. Gravity's lensing of light is responsible for the phenomenon of black holes and the distinctive shadow seen in groundbreaking images thereof obtained in the past several years.



This project will guide students through the process by which one can compute bent trajectories of light in GR through numerical integration of the geodesic equation,

$$\ddot{\gamma}^k + {\Gamma_{ij}}^k \dot{\gamma}^i \dot{\gamma}^j = 0,$$

with the goal of visualizing the distortion of simple images by certain astrophysical sources. We will begin with the simplest case of a static Schwarzschild black hole, described by the metric

$$ds^2 = -\left(1 - \frac{2m}{r}\right) dt^2 + \left(1 - \frac{2m}{r}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right),$$

with the constant m being the black hole's mass. In this way, one can corroborate, e.g., the bending angle of light deflected by the sun or the precession rate of the major axis of a planet's elliptical orbit, the first two major successes of GR.

Ultimately, we will adapt to the Vaidya metric,

$$ds^2 = -\left(1 - \frac{2m(v)}{r}\right) dv^2 + 2dvdr + r^2 \left(d\theta^2 + \sin^2(\theta) d\phi^2\right),$$

wherein the mass $m(v)$ is an increasing *function*, which provides a simple model for a *dynamical* black hole whose mass is increasing due to the accretion of matter. Depending on the rate of accretion, this metric can yield a *naked singularity*, a singularity visible to distant observers. Our objective will be to assess whether there are any distinctive lensing features associated to these naked singularities. The existence of naked singularities would have significant implications for the foundations of GR, and indeed the self-containment of classical physics at large, and so it is of great interest to establish observational features by which they might be recognized.

Prerequisites are simply comfort with differential equations and multivariable calculus. Some knowledge of differential geometry and/or relativity would provide deeper insight, but it is not required. In particular, it is alright if you do not yet know how to parse the content of the three equations above.