LO(G)M RESEARCH PLAN: THREE WAVE INTERACTION AND DYNAMIC WAVE TURBULENCE

CHRIS DISCENZA

Imagine tossing a book in the air and observing how it rotates. Consider flipping the book about axes that are perpendicular to the rectangular sides. If the dimensions of the book are all unequal to each other then tossing it about one of the axes will exhibit unstable behavior. The initial rotation about that axis will twist into a rotation about one of the other two axes. However, rotation about the other two axes will be stable throughout the toss. It turns out the the unstable axis is the with a moment of inertia between those of the other two axes. Rigid body rotation is an example of a nonlinear system described by three coupled partial differential equations and this phenomenon is known as *modulational instability.* This can be represented as a diagram where the magnitude of rotation about each axis is the vertex of a triangle. Then we can view the modulational instability as a directed graph on the triangle.

We can extend this system beyond three vertices to an arbitrarily large cluster of connected triangles. Such systems are used to model nonlinear three-wave interactions of plasma waves or water waves. The vertices now represent the wave modes which are amplitudes associated to a frequency-wavenumber pair. The dominate wave interactions occur between three modes that satisfy a set of resonance conditions. The dynamics is then described by the cluster of connected triangles representing the resonant modes. Given some initial conditions for each mode, the topology of the cluster will determine a cascade of modulation instability. We will explore the modulational instability cascade of various connected clusters of resonant modes and the effects of forcing and dissipation. This is small glimpse into the somewhat new field referred to as *dynamic turbulence.*