1 Large Scale geometry of Integers

Geometric group theory is a fascinating area of mathematics that explores groups through the geometric spaces on which they act. A fundamental example is the group of integers, $\mathbb{Z}$. Despite its simplicity, the geometric structure of $\mathbb{Z}$ provides intriguing insights and serves as a foundational model for more complex topics in the field.

In this project, we aim to delve into the quasi-isometry and rough isometry of $\mathbb{Z}$. These concepts will help us understand how $\mathbb{Z}$ can be compared to other geometric structures. Additionally, if time permits, we will analyze the general left-invariant metrics on $\mathbb{Z}$ and introduce broader concepts related to nilpotent, solvable, and hyperbolic groups. This exploration will provide a comprehensive understanding of how geometric properties can illuminate the nature of various groups.

Preliminary: No. But Math 312 or an equivalent class covering basic group theory is welcome, but not needed (only the general picture part needs some background for basic group theory). Any analysis class similar to Math 351 covering liminf and limsup will be great, it will only be used in one place.